NUMERICAL EXAMPLES IN HEAT

R. E. DAY.

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BY

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PREFACE.

IT is not uncommon for the beginner in Physics to experience considerable difficulty in the application of the principles of Science to the solution of problems in which a definite numerical result is asked for. Considering that a knowledge of physical science which cannot be reduced to concrete numbers is no real knowledge at all, it is very important that, at an early stage of his work, the student should have plenty of practice in applying the facts and theories which he reads about in his text-book, to the numerical solution of such questions as are of frequent occurrence in the practical applications of the subject.

This little book is intended to assist those who are studying Heat in acquiring readiness in solving such problems. I should advise the student who uses it to try for himself those examples which are worked out in full before examining their solutions. Even if he fails to obtain correct results his time will not have been wasted, for he will have to be constantly going back to his text-book for the elucida-

tion of particular points, and the care and attention to detail thus rendered necessary will materially assist him in acquiring a firm hold of the principles of the subject.

My friend Mr. C. D. Webb has kindly assisted me in checking the numerical results, but I shall feel indebted to any student who will point out to me any inaccuracies which may have escaped our notice.

R. E. DAY.

48 BELSIZE SQUARE, N.W.

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NUMERICAL EXAMPLES IN HEAT.

THERMOMETRIC SCALES.

(1.) The scale of a thermometer between the freezing and the boiling points of water is 120 millimetres long. What is the length of each degree (*a*) on the Centigrade, (*b*) on the Fahrenheit, and (*c*) on the Réaumur scale?

(*a*) In the Centigrade thermometer the freezing point of water is marked 0°, and the boiling point 100°, and therefore the length of a Centigrade degree on this thermometer is

$$\text{C.} = \frac{120}{100} = 1 \cdot 2 \text{ millimetre.}$$

(*b*) In Fahrenheit's thermometer the freezing point of water is marked 32°, and the boiling point 212°, and therefore the space between the boiling and the freezing points contains 180 Fahrenheit degrees. Hence the length of a Fahrenheit degree on this thermometer is

$$\text{F.} = \frac{120}{180} = \frac{2}{3} \text{ millimetre.}$$

(*c*) In Réaumur's thermometer the freezing point is marked 0°, and the boiling point 80°; hence the length of a Réaumur's degree on this thermometer is

$$\text{R.} = \frac{120}{80} = 1 \cdot 5 \text{ millimetre.}$$

(2.) In another thermometer the distance between the boiling and the freezing points is 144 millimetres. What is the length of each degree on the same three scales?

Answer. 1·44, ·8, 1·8 millimetre.

(3.) The length of a Fahrenheit degree on a sensitive thermometer is 20 millimetres. What would be the lengths of a Centigrade and a Réaumur degree?

Answer. C.=36 millimetres; R.=45 millimetres.

(4.) The scale of a *fractional* thermometer which ranges from 95° to 105° C. is 27 centimetres long. Find the length of a Fahrenheit and of a Réaumur degree on this thermometer.

Answer. F.=15 millimetres; R.=33·75 millimetres.

Note.—In many cases, e.g. in the construction of clinical thermometers, a minute subdivision of the scale over a limited range of temperature is required. When the whole scale of a thermometer embraces only a small fraction of the range of temperature between the boiling and freezing points of water the thermometer is called a *fractional* one, and as the range is generally very *open*, minute variations of temperature between the given limits can be easily observed.

(5.) Assuming the mean temperature of the air to be 59° Fahrenheit, what are the corresponding numbers on the Centigrade and Réaumur scales?

For the conversion of 'readings' from one of these scales to either of the other two we have the following equations:

$$\text{F.} - 32 = \frac{9}{5}\text{C.} = \frac{9}{4}\text{R.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{C.} = \frac{5}{4}\text{R.} = \frac{5}{9}(\text{F.} - 32) \quad . \quad . \quad (2)$$

$$\text{R.} = \frac{4}{5}\text{C.} = \frac{4}{9}(\text{F.} - 32) \quad . \quad . \quad (3)$$

In the present case we use equations (2) and (3), and we get

$$C.=\frac{5}{9}(59-32)=\frac{5\times 27}{9}=15\,;$$

$$R.=\frac{4}{9}(59-32)=\frac{4\times 27}{9}=12\,;$$

(6.) Find the corresponding temperatures on the Centigrade scale of the following melting points:

Lead, 626° F.; bismuth, 508° F.; tin, 442° F.; Rose's metal, 200° F.

Answers. 330° C.; $264\frac{4}{9}$° C.; $227\frac{7}{9}$° C.; $93\frac{1}{3}$° C.

Note.—Rose's metal is an alloy of the first three, and the student should here notice that the melting point of an alloy is generally lower than that of any of its constituents.

(7.) Convert the following mean temperatures on the Centigrade scale to their equivalents on the Fahrenheit scale:—

Place.	Winter.	Summer.	Range.
Shetland . .	4·05° C.	11·92° C.	7·87° C.
Moscow .	−10·22° C.	12·55° C.	27·77° C.

Answers.

Shetland . .	39·29° F.	53·46° F.	14·17° F.
Moscow . .	13·6° F.	54·59° F.	40·99° F.

(8.) Find the equivalents on the Réaumur scale of the following temperatures:—

Usual temperature of the human body . .	98·6° F.
" a common frog . .	64° F.
" a chicken	111° F.

Answers. 29·6° R.; $14\frac{2}{9}$° R.; $35\frac{1}{9}$° R.

(9.) Water has its greatest density at the temperature of about 39·2° F. What is the corresponding 'reading' on the Centigrade scale? *Answer.* 4° C.

(10.) In the expedition to China in 1839 the Russian

army experienced for several days a temperature of $-32·8°$ R. What would this be on Fahrenheit's scale?

Answer. $-41·8°$ F.

(11.) Assuming that the absolute zero of the thermodynamic scale is $-273·7°$ Centigrade, what is this on Fahrenheit's scale? *Answer.* $-460·66°$ F.

(12.) In De Lisle's thermometer (which is used in Russia) the boiling point of water is marked 0°, and the freezing point 150°. What degree of Fahrenheit corresponds to 135° of De Lisle? *Answer.* 50° F.

(13.) The temperature at which mercury freezes is indicated by the same number on the Centigrade and on Fahrenheit's scale. What is the number?

Let $x =$ the number required, then by equation (1) of Example 5 we have

$$x - 32 = \frac{9x}{5} \therefore x = -40.$$

(14.) At what temperature will the 'reading' on Fahrenheit's scale be twice as great as the corresponding 'reading' on the Centigrade scale?

Let $x°$ F. be the required temperature, then the corresponding reading on the Centigrade scale is $\frac{5}{9}(x-32)$, and by the conditions of the problem we have

$$x = 2 \times \frac{5}{9}(x-32) \therefore x = 320° \text{ F.}$$

(15.) Find the temperature for which the 'reading' on a Fahrenheit thermometer is 9 times as great as the corresponding 'reading' Centigrade. *Answer.* 40° F.

(16.) At what temperature is the sum of the readings on the R., C., and F. scales equal to 212?

Let $x =$ the required temperature on the Centigrade scale, then the corresponding readings on the F. and R. scales will be F. $= 32 + \frac{9x}{5}$ and R. $= \frac{4x}{5}$, and by the con-

ditions of the problem we have

$$\frac{4x}{5} + x + 32 + \frac{9x}{5} = 212$$

$$\therefore x = 50^\circ \text{ C.}$$

(17.) The sum of the readings of a thermometer graduated both on Fahrenheit's and the Centigrade scale is 60. What is the temperature? *Answer.* 10° C.

(18.) The difference of the readings of the same thermometer on another occasion was 40. What was the temperature? *Answer.* 10° C.

(19.) One thermometer marks two temperatures by 20° and 28°, and another marks the same two temperatures by 25° and 35°. What will the latter mark when the former marks 36°?

Since $(28-20)$ or 8 degrees of the first thermometer indicate the same rise of temperature as $(35-25)$ or 10 degrees of the second thermometer,

$\therefore$ one degree of first thermometer $= \frac{5}{4}$ of a degree of second thermometer, and $\therefore$ a rise of 16 degrees on the first corresponds to a rise of $\frac{5}{4} \times 16$ or 20 degrees on the second thermometer. Hence when the first thermometer marks 36° the second will mark $(25+20) = 45$.

(20.) A thermometer (A) marks two temperatures by 10 and 12, and another (B) marks the same two by 12 and 15. What will B mark when A marks 16? *Answer.* 21°.

LINEAR EXPANSION.

(1.) A bar of lead whose length at 0° C. was 152·32 centimetres was placed in a Ramsden's trough and heated to 100° C., when its length was found to be 152·76 centimetres. Find the coefficient of linear expansion of lead.

If a bar of any substance have its temperature raised 1° C., then the fraction expressing the ratio of the increment

of length to its original length is called the coefficient of linear expansion of the substance.

In the present case the increment of length for a rise of 100° C. in the temperature of the bar is

$$152{\cdot}76 - 152{\cdot}32 = {\cdot}44 \text{ cm.},$$

and if the expansion were uniform between the temperatures 0° C. and 100° C. the coefficient of linear expansion for 1° C. would be

$$= \frac{{\cdot}44}{152{\cdot}32 \times 100} = {\cdot}0000289.$$

Note.—The student will find this apparatus fully described in the 'Phil. Trans.,' vol. lxxv. The sensibility was such that the observations of the length of the bar were known to be correct to a millionth part.

(2.) In one of General Roy's experiments ('Phil. Trans.,' vol. lxxv.) a rod of steel which was 5 feet long at 0° C. was 60·068684 inches long at 100° C. Find the mean coefficient of linear expansion for 1° C. for this particular quality of steel. *Answer.* ·00001145.

(3.) A bar of zinc 1·97 metre long at 0° C. was laid in the middle trough of a Ramsden's apparatus, and when it was heated to 100° C. its length was found to be 1·976676 metre. Find the coefficient of linear expansion of zinc.

Answer. ·00003388.

(4.) The length of a brass bar at the temperature of melting ice was 60·2 inches, and at the temperature of boiling water it was 60·31 inches. Find the mean coefficient of linear expansion for 1° F. *Answer.* ·00001015.

(5.) A steel bar which was 400 centimetres long at 0° C. was found to be 400·5 centimetres long at 100° C. Find the mean coefficient of linear expansion of steel for 1° C.

Answer. ·0000125.

(6.) A copper rod which was 1·5 metre long at 0° C. was found to have increased in length by 2·6 millimetres,

owing to a rise of 100° C. in its temperature. Find the coefficient of linear expansion of copper.

Answer. ·0000173.

(7.) A bar of iron 6·5 inches long at 32° F. was placed in a Daniell's pyrometer and heated in a furnace to 572° F., and its length was then found to be 6·522 inches. Find the mean coefficient of expansion for 1° F.

Answer. ·00000627.

(8.) The length of an iron rail at 15° C. is 30 feet. What will be its length at 10° C. and at 20° C.? The coefficient of expansion of this iron for 1° C. is $\frac{1}{81900}$.

Let l_t represent the length of the bar at any temperature $t°$ C., and let its coefficient of linear expansion for 1° C. be represented by a; then we have the general formula

$$l_t = l_0 (1 + at),$$

$$\text{and } \therefore \quad \frac{l_{t'}}{l_t} = \frac{1 + at'}{1 + at};$$

$$\text{hence } l_{10} = 30 \, \frac{1 + \frac{10}{81900}}{1 + \frac{15}{81900}} = 30 \times \frac{81910}{81915} = 29{\cdot}998 \text{ feet,}$$

$$\text{and } l_{20} = 30 \times \frac{81920}{81915} = 30{\cdot}002 \text{ feet nearly.}$$

(9.) The length of an iron telegraph wire at 13° C. is 220 miles. What will be its length at −15° C., and also at +25° C.?

Answers. 219 miles 1627·7 yards } nearly.
220 „ 56·73 „ }

(10.) A glass scale and a brass one are each one metre ong at 10° C. What will be the difference in their lengths at 100° C.? *Answer.* ·91 millimetre.

(11.) What must be the length of a brass rod at 15° C. in order that at 0° C. it may be exactly two metres long, the coefficient of expansion being ·0000189?

By the method of Example 8 we have

$$l_{15} = l_0 (1 + at) = 2 (1 + 15 \times \cdot 0000189)$$
$$= 2 \cdot 00057 \text{ metres nearly.}$$

(12.) The old 'standard yard of 1760' was the distance between two marks on a certain brass rod at 62° F. What was this distance at the temperature of 212° F.?

Answer. 36·056 inches.

(13.) What must be the length of an iron rail at 20° C. so that it may be exactly 30 feet long at 12° C.? (For coefficient of expansion see Table, p. 14.)

Answer. 30 feet ·032 inch.

(14.) Assuming that the maximum temperature in the sun of a 30-foot cast-iron rail is 124° F., and that the temperature of the air at the time of laying the rail is 50° F., what must be the minimum distance apart of the adjacent ends of two consecutive rails? *Answer.* ·1665 inch.

Less than this distance would endanger the line by the possible *buckling* of the rails. Too great a distance between the rails would make the railroad rough and cause the ends of the rails to wear out rapidly.

(15.) The length of an iron boiler at 0° C. is 18 feet. What will be the increase of its length if its temperature be raised to 170° C.? *Answer.* ·413 inch.

(16.) The keel of an iron steamship is 400 feet long when it is in water which is at 3° C. What will be the change in its length when it is at a place in the tropics where the temperature of the water is 28° C.?

Answer. 1·35 inch.

(17.) The length of an iron girder-bridge at 0° C. is 200 feet. What will be its length when, by the heat of the summer sun, its temperature has risen to 40° C.?

Answer. 200·09 feet.

(18.) What will be the length of the same bridge in winter when its temperature is −20° C.?

Answer. 199·96 feet.

(19.) The distance by rail from San Francisco to Omaha, on the Missouri, is 1,914 miles. Assuming that the average variation of temperature throughout the year is 50° C., what is the variation in the total length of the rails?

Answer. 1·07662 mile.

(20.) The railroad from Sacramento to Kansas city is 2,002 miles long. How much must be allowed for the expansion of the rails if the average annual range of temperature is 48° C.? *Answer.* 1·08108 mile.

(21.) Four scales were constructed of brass, cast iron, copper, and platinum respectively, and they were all exactly one metre long at 0° C. Find their lengths at 25° C. to the nearest hundredth of a millimetre.

Answer.	Brass	= 1000·47	millimetres.
	Iron	= 1000·28	"
	Copper	= 1000·43	"
	Platinum	= 1000·22	"

(22.) An iron bar is 1·32 metre long at 0° C. What must be the length of a brass rod at 0° C. so that for any given change of temperature the expansion of the two rods may be equal?

Let x = required length of brass rod in metres,
a_1 = coefficient of expansion of brass,
a_2 = " " " iron;
then $x \times a_1 = 1{\cdot}32 \times a_2$;

$$\therefore\ x = 1{\cdot}32 \times \frac{a_2}{a_1} = 1{\cdot}32 \times \frac{1225}{1875} = {\cdot}8624 \text{ metre},$$

∴ length of brass rod at 0° C. = 862·4 millimetres.

(23.) The secondary wire of Spottiswoode's great induction coil contains 280 miles of copper wire, wound in 340,000 turns. If the temperature change by 30° C., express in 'turns' the variation in the length of the wire.

One turn $= \frac{280}{340000}$ mile;
expansion $= 280 \times 30 \times {\cdot}00001715$ mile;

$$\therefore \text{ variation in length} = \frac{280 \times 30 \times {\cdot}00001715 \times 340000}{280} \text{ turns.}$$
$$= 175 \text{ turns nearly.}$$

(24.) The secondary wire of another induction coil is of copper, and is 60 miles long at 10° C. What will be its length at 25° C.?

Answer. 60 miles 27 yards 6 inches nearly.

(25.) A brass bar and an iron bar are of exactly the same length at 100° C., and when placed end to end the sum of their lengths at 0° C. is one metre. The coefficient of expansion of iron being ·0000122, and that of brass ·0000189, find the length of each bar at 0° C.

Let x = length of iron bar at 0° C. in metres,

then $1 - x$ = " brass " " "

then if a_1 = coefficient of expansion of iron, and a_2 that of brass,

$$x(1 + 100a_1) = (1 - x)(1 + 100a_2),$$

$$\therefore \frac{x}{1-x} = \frac{1+100a_2}{1+100a_1} = \frac{1\cdot00189}{1\cdot00122} = 1\cdot00067 \text{ nearly};$$

$$\therefore x = \cdot50017 \text{ metre nearly};$$

∴ length of iron bar = 500·17 millimetres nearly,

" brass " = 499·83 " "

(26.) The length of a brass rod at the temperature of 15° C. is two metres. What must be the length at 0° C. of an iron bar so that at 30° C. these two bars may be of exactly the same length? *Answer.* 1·53 metre.

(27.) A bar of platinum is 1·82 metre long at 0° C. What must be the length at 0° C. of a brass bar so that if the two bars be heated to 100° C. they may be of exactly the same length? *Answer.* 1·8181 metre.

(28.) What should be the length at 0° C. of the brass bar so that the two bars may be of the same length at 300° C.?

Answer. 1·8146 metre nearly.

(29.) If the bars mentioned in Example 28 were riveted together at one extremity but free at every other point, what would be the distance between their free ends at 20° C.?

Answer. 5 millimetres nearly.

(30.) If the difference in length of the two bars was 5·13 millimetres, what was their temperature?

Let $x°$ C. = required temperature; then
length of platinum at $x°$ C. $= 1{\cdot}82\ (1 + x \times {\cdot}00000875)$,
" brass " $= 1{\cdot}8146\ (1 + x \times {\cdot}00001875)$;

$$\therefore 1{\cdot}82\ (1 + x \times {\cdot}00000875) - 1{\cdot}8146\ (1 + x \times {\cdot}00001875) = {\cdot}00513,$$

whence $x = 15$ nearly.

Note.—This example illustrates the use of Borda's pyrometric standard measure, which was employed by him in measuring the great arc of the meridian in France. By this arrangement the measuring bar acted as a thermometer which indicated its own temperature.

(31.) The apparent length of a wire when measured with a brass scale at 15° C. was 22·735 metres. If the brass scale was correctly graduated at 0° C., what was the real length of the wire?

The *apparent* length is the length indicated by the number of graduations of the scale. As the scale expands with the heat, the absolute value of each subdivision is greater than its indicated value.

At 15° C. the true length of each *apparent* centimetre of the scale is $1 + 15 \times {\cdot}00001875 = 1{\cdot}00028125$ centimetre, and if x represent the true length of the wire in centimetres

$$x = 2273{\cdot}5 \times 1{\cdot}00028125 = 2274{\cdot}14 \text{ centimetres.}$$

(32.) A platinum metre scale was correct at 0° C. The *apparent* length of a rail at 30° C. was, according to this scale, 10·278 metres. What was its true length?

Answer. 10·2807 metres nearly.

Note.—This correction for the expansion of the scale is practically insensible for ordinary changes of temperature and for short lengths.

(33.) A yard denotes the length at $16\frac{2}{3}$° C. of a certain standard brass bar, and the metre is the length at 0° C. of a certain standard platinum bar. It is known that one metre

is equal to 39·370432 inches. Compare the lengths of the two bars at 27° C. and at 0° C.

Answer. Ratio at 27° = 1·09367.
" 0° = 1·09396.

(34.) The length of wire between a distant signal and a signal box is 800 yards, and the coefficient of expansion of the wire for 1° C. is $\frac{1}{81900}$. Assuming that the wire has to be lengthened 4 inches to bring the signal to 'danger,' find what fall of temperature would lower the signal from 'danger' to 'clear.'

Let x = required change of temperature;

$$\text{then } \frac{x \times 800 \times 36}{81900} = 4, \text{ whence } x = 11\tfrac{3}{8}° \text{ C.}$$

(35.) At a given temperature an iron pendulum of a certain length beats seconds. The coefficient of expansion of the iron being ·0000118, find the diminution in the number of vibrations per day when the temperature has risen 20° C.

The time of oscillation of a pendulum at a given place varies directly as the square root of its length. Hence if l_1, t_1, and l_2, t_2 represent the lengths and corresponding times of oscillation, we shall have

$$l_2 = l_1 (1 + 20a),$$

$$\therefore \frac{t_2}{t_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1 + 20a};$$

and if n_1 and n_2 be the numbers of oscillations per day,

$$\frac{n_1}{n_2} = \frac{t_2}{t_1} = \sqrt{1 + 20a} = \sqrt{1·000236} = 1·000118.$$

$$\text{But } n_1 = 3600 \times 24; \quad \therefore n_2 = \frac{3600 \times 24}{1·000118} = 86389·8\,;$$

$$\therefore n_1 - n_2 = 86400 - 86389·8 = 10·2.$$

(36.) A clock has a brass pendulum whose coefficient of expansion is ·0000189. What will be the difference in its

rate per day when the temperature is 0° C. and when it is 30° C.? *Answer.* 24·5 seconds.

(37.) The time of one oscillation of an iron pendulum is 1·434 second. Find the number of oscillations it will lose per day if the temperature be increased by 15° F., the coefficient of expansion for 1° C. being ·0000122.

Answer. 3 nearly.

(38.) A clock with an iron pendulum beats seconds at a certain temperature. What will be the change in temperature if it loses one second in 24 hours, the coefficient of expansion for 1° C. being $\frac{1}{88889}$?

Let x° C. be the required change in the temperature; then, as in Example 35, we have

$$\frac{n_1}{n_2} = \frac{t_2}{t_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1 + xa}.$$

But $n_1 = 86400$, and $n_2 = 86399$;

$$\therefore\ 1 + xa = \left(\frac{86400}{86399}\right)^2 \text{ and } a = \frac{1}{88889};$$

$$\therefore\ x = \frac{172799 \times 88889}{(86399)^2} = 2{\cdot}0576$$

$$= +\ 2° \text{ C. approximately.}$$

N.B. The length of a seconds pendulum is about 39·14 inches.

(39.) An iron pendulum 4 feet long makes 78,030 oscillations in one day. On another day it is observed to make only 78,021 oscillations in the day. Taking the coefficient of expansion of the iron for 1° C. at ·000012204, find the change of temperature. *Answer.* 18·9° C.

(40.) In consequence of a rise of temperature a seconds pendulum loses 8 seconds in a day. Assuming the coefficient of expansion of the rod for 1° C. to be ·000012, find the change of temperature. *Answer.* 15·4° C. nearly.

(41.) An iron pendulum which makes 30 vibrations in a minute is 156·8 inches long, and its coefficient of expansion

for 1° C. is $\frac{1}{88889}$. What must be the change of temperature if it were to lose 30 seconds in 24 hours?

Answer. 61·79° C. nearly.

(42.) A cast-iron rod 16·5 centimetres long at 15° C. was placed in a Daniell's pyrometer and inserted in a furnace, and after it had acquired the temperature of the furnace its length was found to be 16·59 centimetres. Taking ·0000112 to be the coefficient of expansion, find the temperature of the furnace. *Answer.* 502° C.

Note.—Unless otherwise stated, the following are the coefficients of linear expansion for 1° C. which have been employed in these Examples :—

For cast iron . . .	$\frac{1}{88889}$.
„ copper . . .	$\frac{1}{58309}$.
„ platinum . .	$\frac{1}{114285}$.
„ brass . . .	$\frac{1}{53333}$.
„ lead . . .	$\frac{1}{35026}$.
„ glass . . .	$\frac{1}{115607}$.

COMPENSATING PENDULUMS.

(1.) A small heavy bob C is attached to the lower end of a thin iron wire CBA (see fig. 1) which passes freely through a small hole in the base of a vertical rectangle of brass rods, and is attached to the top at A so that the portion BC can oscillate as a pendulum. What must be the length of AB so that the distance BC may be one yard at all temperatures?

Let a_2 and a_1 be the coefficients of expansion of brass and iron for 1° C., and the length of ABC at 0° C. be l feet, that of AB $= x$ feet.

Then at 0° C. we shall have

$$l - x = 3 \quad \ldots\ldots\ldots \quad (1)$$

and at any other temperature t° C. we shall have

$$l(1+a_1 t)-x(1+a_2 t)=3 \quad . \quad . \quad . \quad (2)$$

From these two equations we get

$$la_1 = xa_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and eliminating l between equations (1) and (3)

$$x=\frac{3a_1}{a_2-a_1}$$

and $\left.\begin{array}{l} a_1 = \cdot 0000122 \\ a_2 = \cdot 0000188 \end{array}\right\} \therefore \; x=\frac{3 \times \cdot 0000122}{\cdot 0000066}=5\tfrac{6}{11}$ feet.

(2.) A pendulum is constructed of a glass rod 3 feet long, having a small flange at the bottom upon which rests a heavy leaden disc. What must be the radius of the disc so that the distance between the point of suspension of the rod and the centre of the disc may not vary with changes of temperature?

Let l = length of the rod at 0° C.
r = radius of disc ,,
a_1 = coefficient of expansion of glass
a_2 = ,, ,, lead;

then (see fig. 2) at 0° C.

$$\text{AC}=\text{AB}-\text{BC}=l-r \quad . \quad . \quad . \quad . \quad (1)$$

and at t° C

$$\text{AC}=l(1+a_1 t)-r(1+a_2 t) \quad . \quad . \quad . \quad (2)$$

From equations (1) and (2) we get

$$0 = la_1 t - ra_2 t$$

$$\therefore \; r=l\frac{a_1}{a_2}=\frac{3 \times \cdot 00000865}{\cdot 00002855} \text{ foot}$$

$$= 10\cdot 9 \text{ inches approximately.}$$

(3.) A Graham's mercurial pendulum consists of a glass cylinder containing mercury which is suspended by a steel

rod 3 feet long, the lower end of which has a flange supporting the glass cylinder. The coefficient of expansion of the steel for 1° C. being $\frac{1}{79680}$, and that of the apparent expansion of mercury in glass $\frac{1}{6480}$, what must be the height of the mercury column?

Let x = length of the mercury column in feet
l = " steel rod in feet
a_1 = coefficient of linear expansion of steel
a_2 = " apparent " mercury in glass.

When the temperature rises t° C. the centre of mass of the mercury will be *lowered* $a_1 l$ feet in consequence of the expansion of the steel rod, and will be *raised* $\frac{x}{2} \times a_2$ in consequence of the expansion of the mercury, and if these two effects are to neutralise each other we must have

$$\frac{x}{2} \times a_2 = a_1 l$$

$$\therefore \quad x = \frac{a_1}{a_2} \times 2l = \frac{6480}{79680} \times 2 \times 3$$

$$= \frac{81}{166} \text{ foot} = 5{\cdot}86 \text{ inches nearly.}$$

The diameter of the mercury cylinder must be sufficiently great that the centre of oscillation of the whole pendulum may be very near the middle of the mass of mercury.

This pendulum was invented by Graham in 1721.

(4.) Another Graham's pendulum consisted of a glass rod terminating in a glass cylinder containing mercury. The coefficient of expansion of the glass for 1° C. being $\frac{1}{116100}$, and the whole length of the pendulum 5 feet, what would be the requisite height of the mercury in the cylinder? *Answer.* 6·7 inches nearly.

(5.) An iron tube A B (see fig. 3) has a zinc tube inside it which is rigidly attached to a flange at the bottom. From

the top of the zinc tube, and coaxial with it, there hangs an iron rod C D which carries at its lower end a heavy bob. If the length of A B be 3 feet and of C D 3 feet 6 inches, what must be the length of the zinc tube so that the distance A D may remain constant?

Let a_1 = coefficient of expansion of iron
a_2 = " " zinc;

then for compensation we must have

$$(AB + CD)a_1 = BC \times a_2$$

$$\therefore \; BC = \frac{a_1}{a_2} \times 6{\cdot}5 \text{ feet}$$

$$= \frac{1125 \times 6{\cdot}5}{3389} = 2{\cdot}158 \text{ feet.}$$

(6.) 'Smeaton's' pendulum has a solid glass rod with a shoulder at the bottom upon which rests a coaxial tube of zinc a foot long which expands upwards. This is enclosed in an iron tube a foot long with a flange at the top by which it rests on the zinc and expands downwards. At the bottom of the iron tube is a shoulder upon which rests a coaxial lead tube a foot long. What must be the length of the glass rod so that with these three massive tubes forming the bob there may be compensation?

Answer. 5 feet 11 inches very nearly.

Note.—This construction is easy and cheap, but the objection to it is that the parts the expansions of which are opposed to each other are not equally exposed to the external air, and are apt to be at somewhat different temperatures.

(7.) 'Reid's' compensation pendulum has a long central iron rod descending considerably below the bob. Upon a flange at the lower end of the iron rod rests a zinc tube, and the bob is supported on the top of the zinc tube. If the distance between the point of suspension and the bob is to be always 3 feet, what must be the respective lengths of the iron rod and zinc tube at 0° C.?

Let l_1 = length of iron rod at 0° C. in feet
l_2 = „ zinc tube „ „
a_1 = coefficient of expansion of iron
a_2 = „ „ zinc;

then at 0° C. $l_1 - l_2 = 3$ . . . . (1)

at t° C. $l_1(1+a_1t) - l_2(1+a_2t) = 3$ . . . (2)

$$\therefore \frac{l_1}{l_2} = \frac{a_2}{a_1} \quad \ldots \ldots \quad (3)$$

and from (1) and (3) we get

$$l_1 = \frac{a_2}{a_2 - a_1} \times 3 = \frac{3389}{3389 - 1125} \times 3 = 4{\cdot}49 \text{ feet}$$

$$l_2 = \frac{a_1}{a_2 - a_1} \times 3 = 1{\cdot}49 \text{ feet.}$$

Note.—Captain Kater's pendulum of 1808, which was older than this one, differed from it only in having a wooden rod.

(8.) A steel rod attached to and descending from the suspension is partially enclosed by a zinc tube 25 inches long which rests on a flange at the lower end of the rod. A steel tube with an inner flange at the top is supported by and encloses the zinc tube, and to the lower end of the steel tube a heavy bob is attached. The coefficient of expansion of zinc being about 2·7 times that of steel, what must be the whole length of the steel rod and steel tube so that there may be compensation? *Answer.* 67·5 inches.

(9.) A 'gridiron' pendulum consists of a long rectangle of iron rods suspended by the centre of its upper side. The lower side has a hole in the centre through which passes an iron rod carrying the bob, and this rod is suspended from a cross-piece which is supported on two vertical bars of brass, whose lower ends are rigidly attached to the lower side of the iron rectangle. If the distance between the point of suspension and the centre of oscillation is to be 99·42 centi-

metres and to be invariable, what must be the lengths of the iron and brass rods?

The student will notice that the 'gridiron' pendulum is a particular case of a tube pendulum such as that described in Example 5, and that a longitudinal section of such a pendulum would form a 'gridiron' with the advantage that all parts would be equally acted upon by the external air.

Let l_1 = length of iron rectangle + central iron rod
l_2 = " one of the brass rods
a_1 = coefficient of expansion of iron
a_2 = " " brass;

then, as before, we have

$$l_1 - l_2 = 99{\cdot}42 \quad . \quad . \quad . \quad (1)$$
$$l_1(1 + a_1 t) - l_2(1 + a_2 t) = 99{\cdot}42 \quad . \quad . \quad . \quad (2)$$
$$\therefore \frac{l_1}{l_2} = \frac{a_2}{a_1} \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{Hence } l_1 = \frac{a_2}{a_2 - a_1} \times 99{\cdot}42 = \frac{1875}{1875 - 1225} \times 99{\cdot}42$$
$$= \frac{1875}{650} \times 99{\cdot}42 = 286{\cdot}79 \text{ centimetres nearly}$$
$$l_2 = 187{\cdot}37 \text{ centimetres.}$$

(10.) What would have been the values of l_1 and l_2 if zinc had been used instead of brass?

Answer. 155·7 and 56·28 centimetres.

Note.—The length of a seconds pendulum anywhere in England is about 99·42 centimetres.

(11.) At the Paris Exhibition of 1855 there was a compensating pendulum which consisted of a jointed rhombus of iron rods each 2 feet long with a horizontal diagonal of brass. What would have to be the length of this diagonal so that if the rhombus were suspended from one free corner the distance between that and the lower one should not vary with the temperature?

Let A (see fig. 4) be the upper corner, C the lower one,

BD the horizontal diagonal and E its intersection with the vertical through A and C.

Also let a_1 = coefficient of expansion of iron

a_2 = " " brass;

then by Euclid I., Prop. 47, we have

$$\text{At } 0^\circ \text{ C.} \quad AE^2 = AB^2 - BE^2$$

$$\text{and at } t^\circ \text{ C.} \quad AE^2 = AB^2(1+a_1t)^2 - BE^2(1+a_2t)^2$$

$$= AB^2 - BE^2 + 2t(AB^2a_1 - BE^2a_2) \text{ nearly.}$$

$$\text{Hence } \frac{AB}{BE} = \sqrt{\frac{a_2}{a_1}}$$

$$\therefore \text{ diagonal} = 2BE = 2AB \times \sqrt{\frac{a_1}{a_2}} = 4 \times \sqrt{\frac{1225}{1875}}$$

$$= 3{\cdot}233 \text{ feet}$$

$$= 3 \text{ feet } 2{\cdot}8 \text{ inches nearly.}$$

SUPERFICIAL EXPANSION.

(1.) The coefficient of linear expansion of lead being $\frac{1}{35026}$ for 1° C., and a square sheet of lead having its temperature raised from 0° C. to 100° C., what error will be committed by assuming that the coefficient of superficial expansion is twice that of linear expansion?

Let A = original area of sheet of lead in square centimetres

A′ = expanded area of sheet of lead in square centimetres

a = original length of one side in centimetres

α = coefficient of linear expansion;

then the length of a side at 100° C. is $a(1+100\alpha)$, and $\therefore$ the area at 100° C. is

$$A' = a^2(1+100\alpha)^2 = A\{1+200\alpha+10000\alpha^2\}$$

$$= A\left\{1 + \frac{200}{35026} + \frac{10000}{(35026)^2}\right\}$$

$$= A\{1 + {\cdot}00571004 + {\cdot}00000815\}$$

$$= A \times 1{\cdot}00571819.$$

As far as the seventh place of decimals the error committed by neglecting the third term is

$$A \times \cdot 0000082,$$

and if the original area had been 1 square metre the error would have been ·082 of a square centimetre.

(2.) A square metre of cast iron is heated from 0° C. to 100° C. Find the error committed in calculating the expanded area by taking the coefficient of superficial expansion as equal to twice that of linear expansion, which is $\frac{1}{88889}$ for 1° C. *Answer.* ·0127 of a square centimetre.

Note.—These examples show that for all ordinary purposes the coefficient of superficial expansion may be taken to be sensibly equal to twice the coefficient of linear expansion. The error thus incurred is much less than the limit of experimental error in the determination of the value of the coefficient of linear expansion.

(3.) The edge of a square sheet of lead expands when heated in the ratio of 1 : 1·0028. Find its superficial expansion both accurately and approximately.

By the method of Example 9 we find that

$$\frac{\delta A}{A} = 2 \times \cdot 0028 + \cdot 00000784 \text{ accurately,}$$
$$= 2 \times \cdot 0028 \text{ approximately.}$$

(4.) At 10° C. a plate of cast iron has an area of 15 square feet. What will be its area at 20° C. and at 0° C. respectively?

Let A_t = area of the cast-iron plate at t° C. and a = coefficient of linear expansion; then, since the coefficient of superficial expansion is equal to twice the coefficient of linear expansion,

$$A_t = A_0(1 + 2at),$$

$$\therefore A_0 = \frac{A_t}{1+2at} = \frac{A_{10}}{1+\frac{2 \times 10}{88889}} = \frac{15 \times 88889}{88909}$$

$$= 14 \cdot 9966 \text{ square feet.}$$

Also $A_{20} = A_{10} \times \frac{88929}{88909} = 15{\cdot}0034$ square feet.

(5.) The area of some lead roofing at 10° C. is 400 square feet. What will be the increase in area if the temperature rises to 30° C.? *Answer.* 65·66 square inches.

(6.) The heating surface of the copper tubes of a boiler is 1,200 square feet at 10° C. What will be their area when the temperature of the tubes has risen to 154° C.?

Answer. 1205·9 square feet nearly.

(7.) A cylindrical iron boiler terminated by plane ends is 16 feet long and 4 feet 6 inches in diameter at 13° C. What will be the increase in its surface when the temperature rises to 100° C.?

With the notation employed in Example 4 we have

$$\begin{aligned} A_{13} &= 2 \times \pi \times (2{\cdot}25)^2 + 2\pi \times 2{\cdot}25 \times 16 \\ &= 258{\cdot}04 \text{ square feet.} \end{aligned}$$

$$\text{Also } A_{100} = A_{13} \frac{1 + \frac{2 \times 100}{88889}}{1 + \frac{13 \times 2}{88889}} = 258{\cdot}04 \times \frac{89089}{88915}$$

$$= 258{\cdot}51 \text{ square feet}$$

$$\therefore \delta A = 258{\cdot}51 - 258{\cdot}04 = {\cdot}47 \text{ of a square foot}$$

$$= 67{\cdot}68 \text{ square inches.}$$

(8.) The diameter of a brass disc at 0° C. is 100 centimetres. By how much will the area of the disc be increased if the temperature be raised to 50° C.?

Answer. 14·73 square centimetres nearly.

(9.) Assuming that the contraction of cast iron in cooling is $\frac{1}{120}$th of its length, what must be the diameter of the pattern for the mould of a fly wheel which is to be 20 feet in diameter? *Answer.* 20 feet 2 inches.

Note.—The usual rule is to make the pattern for a fly wheel $\frac{1}{10}$th of an inch on a foot larger than the wheel is required to be.

(10.) The diameter of an iron fly wheel is to be 25 feet. What must be the diameter of the pattern?

Answer. 25 feet $2\frac{1}{2}$ inches.

(11.) The area of the upper surface of a rectangular plate of cast iron at 0° C. is equal to 25 square decimetres. What will be its area if its temperature be raised to 90° C.?

Answer. 25·05 of a square decimetre.

(12.) If the radius of a brass circle at 0° C. be 4 centimetres, by how much is the area of this circle greater at 40° C. than it is at 0° C.?

Answer. 7·54 square millimetres.

(13.) A cast-iron plate at 10° C. has a surface area of 12 square feet. What will be its area at 30° C. and at 0° C. respectively?

Answer. 12·0054 and 11·9973 square feet.

(14.) A brass ball whose diameter is 6 centimetres can easily pass through a circular hole of 6·01 centimetres diameter. Find the range of temperature through which the ball must be heated so as just not to be able to pass through the hole.

Let x° C. = range of temperature required, then

$$6\left(1 + \frac{x}{53333}\right) = 6{\cdot}01$$

$$\therefore x = \frac{{\cdot}01 \times 53333}{6} = 88{\cdot}9^\circ \text{ C. nearly.}$$

(15.) At the Manchester Exhibition of 1857 Sir Joseph Whitworth exhibited an *internal* gauge having a cylindrical aperture of ·5770 inch in diameter, and an *external* gauge consisting of a solid steel cylinder ·5769 of an inch in diameter. This was so loose that it did not seem to fit at all. What increase of temperature would make the external gauge exactly fit the internal one? *Answer.* 13·81° C.

(16.) The dimensions of a rectangular steel bar magnet were as follows: length = 30 centimetres, breadth = 4 centimetres, thickness = ·6 centimetre; and its density

at 0° C. was 7·8. If its temperature changed from 0° C. to 30° C. what would be the change in its moment of inertia about a vertical axis parallel to the thinnest edge, the coefficient of linear expansion for 1° C. being ·000012?

By the method of Example 8, p. 48 of Day's 'Electrical Measurements,' we have

$$K_0 = m \times \frac{x^2+y^2}{12} = 561·6 \times \frac{30^2+4^2}{12} = 42869 \text{ nearly.}$$

After expansion by heat

$$x' = 30\,(1 + 30 \times ·000012) = 30·0108$$
$$y' = 4\,(1 + 30 \times ·000012) = 4·00144,$$
$$\therefore K_{30} = 561·6\,\frac{(30·0108)^2 + (4·00144)^2}{12} = 42900 \text{ nearly}$$
$$\therefore \delta K = 42900 - 42869 = 31.$$

(17.) What would have been the change in the moment of inertia about the same axis if the length of the bar had been 1 metre and its breadth 2 centimetres, all else remaining the same as before? *Answer.* 571·6.

(18.) The dimensions of a cast-iron cylinder at 0° C. are as follows: length = 50 centimetres, diameter = 20 centimetres, and density = 7·2. The coefficient of linear expansion being ·0000122, find the change in its moment of inertia about the axis when the temperature rises to 25° C.

If the moment of inertia of any body about a given axis be denoted by K, and if in consequence of a change of temperature of $t°$ the linear dimensions of the body are altered in the ratio of $1 + at : 1$, then the moment of inertia with respect to the given axis becomes $(1 + at)^2 K$ or approximately $(1 + 2at)K$.

In the present case

$$\delta K = \frac{M}{2} r^2 \times 2at = \frac{\pi \times 10^2 \times 50 \times 7·2}{2} \times 10^2 \times ·00061$$
$$= 3450 \text{ nearly.}$$

(19.) The moment of inertia of a cast-iron fly wheel at 0° C., when expressed in foot pounds, is 50750. What will be the change in its value when the temperature rises to 20° C., the coefficient of expansion being ·0000112?

Answer. 22·7 nearly.

CUBICAL EXPANSION.

(1.) The edge of a cube of lead is 10 centimetres long at 0° C. If the cube be heated to 100° C. and the coefficient of linear expansion of lead for 1° C. be $\frac{1}{35026}$, what error will be made in calculating the new volume of the cube by assuming that the coefficient of cubical expansion is 3 times that of linear expansion?

Let v = original volume of cube in cubic centimetres
v′ = final " " "

$$\text{Then } v' = v\left\{1 + \frac{100}{35026}\right\}^3$$

$$= v\left\{1 + \frac{3 \times 100}{35026} + 3 \times \left(\frac{100}{35026}\right)^2 + \left(\frac{100}{35026}\right)^3\right\}$$

$$= v\,\{1 + {\cdot}00856507 + {\cdot}000024453 + {\cdot}000000023\}$$

$$= v \times 1{\cdot}0085895 \text{ as far as the 7th place of decimals.}$$

But v = 1000 cubic centimetres

∴ error = ·0244 c.c.

(2.) If the cube had been of cast iron, whose coefficient of linear expansion is $\frac{1}{88889}$ for 1° C., what would have been the corresponding error? *Answer.* ·038 c.c.

Note.—These examples show that, as a general rule, the coefficient of cubical expansion may be taken as equal to 3 times that of linear expansion.

(3.) The diameter of a brass ball at 3° C. is 15 centimetres. What will be the volume of the ball at 80° C.?

Let v_t = volume of the ball at $t°$ C.;

$$\text{then } v_{80} = v_0\left(1 + \frac{3\times 80}{53333}\right) = v_0\,\frac{53573}{53333}$$

$$\text{and } v_3 = v_0\left(1 + \frac{3\times 3}{53333}\right) = v_0\,\frac{53342}{53333}$$

$$\therefore\ v_{80} = \frac{53573}{53333} \times \frac{53333}{53342} \times v_3 = \frac{53573}{53342} \times \frac{4}{3}\pi \times (7\cdot 5)^3$$

$$= 1774\cdot 8 \text{ c.c.}$$

(4.) The volume of a cast-iron ball at 10° C. is 2 cubic decimetres. If its temperature be raised to 50° C. what will be its increase of bulk? *Answer.* 2·7 c.c.

(5.) Find the volume of 20 kilogrammes of mercury at 100° C., the coefficient of expansion for 1° C. being $\frac{1}{5550}$ and the density of mercury at 0° C. being 13·59.

Let v_t represent the volume of this mass of mercury at $t°$ C.; then, since the absolute density of a substance is the mass of 1 cubic centimetre of the substance expressed in grammes, we have

$$v_0 = \frac{20000}{13\cdot 59} \text{ c.c.}$$

$$\text{Also } v_{100} = v_0(1 + 100a) = v_0\left(1 + \frac{100}{5550}\right) = v_0 \times \frac{5650}{5550}$$

$$= \frac{20000}{13\cdot 59} \times \frac{5650}{5550} = 1498\cdot 19 \text{ c.c.}$$

(6.) What is the mass of a cast-iron cylinder which at 25° C. is 3·72 metres long and 38 centimetres in diameter, the density of cast iron at 0° C. being 7·207?

$$v_{25} = \pi \times (19)^2 \times 372 \text{ c.c.} = v_0\left(1 + \frac{3\times 25}{88889}\right)$$

$$= v_0 \times \frac{88964}{88889}$$

$$\therefore\ v_0 = \frac{\pi \times (19)^2 \times 372 \times 88889}{88964} = 421536 \text{ c.c.}$$

$$\therefore\ \text{mass} = v_0 \times d_0 = 421536 \times 7\cdot 207 = 3038010 \text{ grammes}$$

$$= 3038\cdot 01 \text{ kilogrammes.}$$

(7.) The density of copper at 0° C. being 8·878, what is the mass of a solid copper ball whose diameter at 80° C. is 16 centimetres? *Answer.* 18·9623 kilogrammes.

(8.) The volume of a copper ball at 25° C. is 1,620 cubic centimetres. What will be its volume at 0° C.?

Answer. 1617·9 c.c.

(9.) The mass of a cast-iron ball is 12 kilogrammes, and the density of cast iron at 0° C. is 7·207. What will be the diameter of the ball at 30° C.?

Let x = radius of the ball in centimetres and d_t represent its density at t° C.; then

$$d_{30} = \frac{7{\cdot}207}{1 + \frac{3 \times 30}{88889}} = \frac{7{\cdot}207 \times 88889}{88979}.$$

Also $12000 = \frac{4}{3}\pi x^3 \times d_{30}$

$$\therefore\ x^3 = \frac{12000 \times 3}{4\pi} \times \frac{88979}{7{\cdot}207 \times 88889}$$

$$\therefore\ x = 7{\cdot}3552 \text{ centimetres}$$

$$\therefore\ \text{diameter} = 14{\cdot}71 \text{ centimetres nearly.}$$

(10.) The density of copper at 0° C. being 8·878, what is its density at 30° C.?

Since $v_0 d_0 = v_t d_t$

$$\therefore\ d_t = d_0 \times \frac{v_0}{v_t} = d_0 \times \frac{1}{1+\frac{3 \times 30}{58309}} = d_0 \times \frac{58309}{58399}$$

$$= 8{\cdot}878 \times \frac{58309}{58399} = 8{\cdot}864.$$

(11.) If the density of cast iron at 0° C. be 7·799 what quantity of iron will there be in 1 cubic decimetre at 25° C.? *Answer.* 7·7924 kilogrammes.

(12.) Assuming that the density of copper at 0° C. with respect to water at its maximum density (4° C.) is 8·88, what will be their relative densities at 20° C.? The density

of water at 20° C. is ·9983, and the coefficient of linear expansion of copper is $\frac{1}{58309}$.

$$\text{Density of copper at } 20^\circ \text{ C.} = \frac{8{\cdot}88 \times 58309}{58369}$$

$$\therefore \frac{\text{density of copper at } 20^\circ \text{ C.}}{\text{density of water at } 20^\circ\text{C.}} = \frac{8{\cdot}88 \times 58309}{{\cdot}9983 \times 58369} = 8{\cdot}886.$$

(13.) The density of brass at 0° C. being 8·383, and that of water at 60° C. being ·983, what will be their relative densities at the latter temperature? *Answer.* 8·4993.

(14.) The density of lead at 0° C. being 11·352, and that of water at 90° C. being ·965, find their relative densities at 90° C. *Answer.* 11·674.

(15.) The specific gravity of a piece of copper which was weighed in water at 30° C. was found to be 8·9. The density of water at 30° C. being ·996, what would be the density of this copper at 0° C.?

$$\text{Mass of 1 c.c. of copper at } 30^\circ \text{ C.} = 8{\cdot}9 \times {\cdot}996$$
$$= 8{\cdot}8644 \text{ grammes.}$$
$$\text{Mass of 1 c.c. of copper at } 0^\circ \text{ C.} = 8{\cdot}8644 \times \frac{58399}{58309}$$
$$= 8{\cdot}878 \text{ grammes.}$$

(16.) A mass of platinum was weighed in water at 20° C., and its specific gravity was found to be 23·055. The density of water at 20° C. being ·9983, and the coefficient of linear expansion of platinum being $\frac{1}{114286}$, find its density at 0° C. *Answer.* 23·027.

(17.) The density of mercury at 0° C. being 13·596, and its coefficient of expansion $\frac{1}{5550}$, what is the volume of 25 kilogrammes of mercury at 100° C.? *Answer.* 1871·9 c.c.

(18.) What space will be occupied at 84° C. by a quantity of oil which exactly measures 1 litre at 0° C., the coefficient of expansion for 1° C. being ·001?

$$v_{84} = v_0 (1 + 84 \times {\cdot}001) = 1{\cdot}084 \times v_0 = 1084 \text{ c.c.}$$

(19.) What will be the space occupied at 75° C. by a quantity of mercury whose bulk at 0° C. is 8 cubic centimetres? *Answer.* 8·108 c.c.

(20.) If the density of absolute alcohol at 0° C. be ·793, what will be its density at 25° C., the coefficient of expansion being ·001? *Answer.* ·774 nearly.

(21.) What is the density of mercury at 20° C. if its density at 0° C. is 13·596? *Answer.* 13·547.

(22.) If 25 cubic centimetres of mercury at 13° C. be heated to 100° C. what will be the bulk?

Answer. 25·391 c.c.

(23.) What must be the internal capacity of a glass flask at 0° C. so that at 25° C. it may just be able to contain 1,200 grammes of mercury?

$$\text{The density of mercury at } 25^\circ \text{ C.} = \frac{13{\cdot}596}{1+\frac{25}{5550}}$$

$$= \frac{13{\cdot}596 \times 5550}{5575}$$

$$\therefore v_{25} = \frac{1200 \times 5575}{13{\cdot}596 \times 5550} \text{ c.c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also

$$v_0 = \frac{v_{25}}{1 + 25a} = \frac{v_{25}}{1+\frac{3 \times 25}{115607}} = v_{25} \times \frac{115607}{115682} \quad . \quad . \quad (2)$$

And substituting the value of v_{25} from (1) in (2) we get

$$v_0 = \frac{115607}{115682} \times \frac{1200 \times 5575}{13{\cdot}596 \times 5550} = 88{\cdot}6 \text{ c.c. nearly.}$$

(24.) A small glass flask is exactly filled by 62·5 grammes of mercury at 28° C. What will be the internal capacity of this flask at 0° C.? *Answer.* 4·617 c.c.

(25.) What is the capacity of a glass flask which is exactly filled by 2·3963 kilogrammes of mercury at 15° C.?

Answer. 176·67 c.c. nearly.

(26.) The internal diameter of a hollow iron ball at 0° C. is 10 centimetres. What mass of mercury can the ball contain at 0° C., and also at 50° C.?

$$\text{Capacity of ball at } 0^\circ \text{ C.} = \frac{4}{3}\pi \times (5)^3 = 523{\cdot}6 \text{ c.c.}$$

$\therefore$ mass of mercury contained by the ball at 0° C.

$$= 523{\cdot}6 \times 13{\cdot}596 = 7118{\cdot}86 \text{ grammes.}$$

Again, the capacity of the ball at 50° C.

$$= 523{\cdot}6\left(1 + \frac{3 \times 50}{88889}\right) = 523{\cdot}6 \times \frac{89039}{88889}.$$

The density of mercury at 50°C.

$$= \frac{13{\cdot}596}{1 + \frac{50}{5550}} = 13{\cdot}596 \times \frac{5550}{5600}$$

$\therefore$ mass of mercury contained by the ball at 50° C.

$$= 523{\cdot}6 \times \frac{89039}{88889} \times 13{\cdot}596 \times \frac{5550}{5600} = 7067{\cdot}21 \text{ grammes.}$$

(27.) What quantity of mercury would this iron ball contain at 100° C.? *Answer.* 7016·47 grammes.

(28.) A lump of copper, whose mass is 500 grammes, is suspended in a vessel of water the temperature of which is 16° C. The density of copper at 0° C. being 8·878, and its coëfficient of linear expansion ·00001715, while the density of water at 16° C. is ·998, find the mass of the water displaced by the copper.

Let x = number of grammes of the water displaced
m = „ „ „ copper
d_t = density of copper at t° C.
δ_t = „ water „

The volume of the copper at t° C. $= \dfrac{m}{d_0}(1 + 3at)$, and

since this is also the volume of the water displaced its mass is $=\frac{m}{d_0}(1+3at)\times\delta_t$

$$\therefore x=\frac{m}{d_0}(1+3at)\times\delta_t=\frac{500}{8\cdot878}(1+3\times16\times\cdot00001715)\times\cdot9989$$
$$=56\cdot3034 \text{ grammes.}$$

(29.) The density of lead at 0° C. is 11·4 and its coefficient of linear expansion is $\frac{1}{35026}$. What will be the apparent loss of weight in a mass of 5·75 kilogrammes of lead when suspended in water at 18° C., the density of water at that temperature being ·9987?

Answer. 504·57 grammes.

(30.) A lump of platinum whose mass is 7·5 kilogrammes is suspended in mercury at the temperature of 25° C. The density of mercury at 0° C. is 13·6, and that of platinum is 22, while the coefficient of cubical expansion of platinum is $\frac{1}{38700}$ and that of mercury $\frac{1}{5550}$. Find the loss of weight.

The volume of platinum at 25° C. $=\frac{7500}{22}\left(1+\frac{25}{38700}\right)$ $=341\cdot13$ c.c., and this is also the volume at 25° C. of the mercury displaced. The corresponding volume at 0° C. would be

$$\frac{341\cdot13}{1+\frac{25}{5550}}=339\cdot6 \text{ c.c.}$$

$\therefore$ the mass of mercury displaced $=339\cdot6\times13\cdot6=4618\cdot56$ grammes.

Hence the apparent loss of weight $=7500-4618\cdot56$ $=2881\cdot44$ grammes.

(31.) A mass of 220 grammes of platinum was suspended from one arm of a balance and counterpoised, and was then immersed in mercury at 0° C., when the loss of weight was found to be 135·96 grammes. The mercury was then slowly heated to 25° C., and the loss of weight was then

found to be 135·439 grammes. The coefficient of expansion of mercury being $\frac{1}{5550}$, and its density at 0° C. 13·596, find the coefficient of cubical expansion of the platinum.

The volume of the mercury displaced at 0° C. $= \frac{135·96}{13·596}$ $=10$ c.c., and this is equal to the volume of the mass of platinum at 0° C.

When the temperature rises to 25° C. the density of the mercury becomes $\dfrac{13·596}{1+\dfrac{25}{5550}}=13·535$, and ∴ the volume of the mercury displaced at 25° C. is $\frac{135·439}{13·535}=10·00659$ c.c.

But this is also the volume of the mass of platinum at 25° C., and ∴ if x represent the coefficient of cubical expansion of platinum for 1° C.

$$10\,(1+25x)=10·00659$$

$$\therefore\ x=\frac{·00659}{250}=·0000263.$$

(32.) The density of a mixture of alcohol and water at 30° C. is ·941, and at 0° C. it is ·961. Find its mean coefficient of expansion for 1° C.

Let x = mean coefficient of expansion for 1° C.; then by the method of Example 10

$$\frac{d_t}{d_0}=\frac{\mathrm{v}_0}{\mathrm{v}_t}=\frac{1}{1+xt}$$

$$\therefore\ x=\frac{d_0-d_t}{t\times d_t}=\frac{·961-·941}{30\times·941}=\frac{·02}{28·23}$$

$$=\frac{1}{1411}\text{ nearly.}$$

(33.) A hollow platinum ball, whose diameter is 5 centimetres, is suspended from one arm of a balance and just floats in mercury at 0° C. If the temperature of the room changes to 25° C. what counterpoise will be required to

prevent the ball from sinking? The density of mercury at 0° C. is 13·596.

The mass of platinum in the ball is equal to the mass of mercury displaced by it at 0° C.; that is, to

$$\frac{4}{3}\pi\times(2\cdot5)^3\times13\cdot596=889\cdot858 \text{ grammes.}$$

The mass of mercury displaced by the ball at 25° C. is

$$=\frac{4}{3}\pi\times(2\cdot5)^3\times\left(1+\frac{3\times25}{114285}\right)\times\frac{13\cdot596}{1+\frac{25}{5550}}=886\cdot448$$

grammes.

Hence the required counterpoise, which is equal to the excess of the mass of the platinum ball over that of the mercury displaced at 25° C., is

$$=889\cdot858-886\cdot448=3\cdot41 \text{ grammes.}$$

(34.) A copper vessel at 100° C. can just hold 1342·4 grammes of mercury, and at 0° C. it can just hold 1359·6 grammes. The coefficient of expansion of mercury being $\frac{1}{5550}$, find the coefficient of cubical expansion of copper.

Since the density of mercury at 100° C. $=\dfrac{13\cdot596}{1+\frac{100}{5550}}$

$$=13\cdot355$$

$$\therefore \text{ capacity of vessel at } 100^\circ \text{ C.}=\frac{1342\cdot4}{13\cdot355}=100\cdot514 \text{ c.c.}$$

$$\text{and the capacity at } 0^\circ \text{ C.}=\frac{1359\cdot6}{13\cdot596}=100 \text{ c.c.}$$

Hence if x be the coefficient of cubical expansion of the copper

$$100\,(1+100x)=100\cdot514$$

$$\therefore x=\frac{\cdot514}{10000}=\cdot0000514.$$

(35.) An iron bottle at 15° C. just holds 18·144 kilogrammes of mercury. When placed in boiling water it is

found that 222 grammes of mercury escape. Find the coefficient of cubical expansion of iron. *Answer.* ·0000334.

(36.) A glass vessel surrounded by melting ice is completely filled by 3·399 kilogrammes of mercury. The vessel is then placed in water which is made to boil, and it is found that 51·5 grammes of mercury escape. Find the coefficient of cubical expansion of this glass. *Answer.* ·0000259.

(37.) A small glass flask is completely filled at 0° C. by 62·5 grammes of mercury. The temperature is then raised to 100° C., and the quantity of mercury which then fills the flask is 61·555 grammes. Find the coefficient of cubical expansion of this glass. *Answer.* ·0000264 nearly.

Note.—This was French glass, with lead in it.

(38.) A B and C D (see fig. 5) are two vertical glass tubes connected at their bases by a U-shaped capillary glass tube B E F D. Mercury was poured into the apparatus, and when the branch A B was kept in melting ice, while the branch C D was kept at 100° C., the difference of level of the mercury in the two branches was found to be 6·3 millimetres, while the surface *m* of the cold mercury was 35 centimetres above the horizontal tube E F. Find the coefficient of absolute expansion of mercury between 0° C. and 100° C.

Let $h_0 =$ height above E F of the surface of the cold mercury
$h_t =$ " " " " hot "
$d_0 =$ density of the cold mercury
$d_t =$ " " hot "
$t =$ temperature of the hot mercury
$x =$ mean coefficient of expansion of mercury between 0° C. and 100° C.

Since the two columns of mercury balance each other their pressures per unit of area at E and F must be equal, and therefore

$$h_0 d_0 = h_t d_t$$

$$\therefore \frac{h_t}{h_0} = \frac{d_0}{d_t} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also, since the density of any given mass of a substance varies inversely with its volume,

$$\frac{d_0}{d_t} = 1 + xt \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equations 1 and 2 we obtain

$$\frac{h_t}{h_0} = 1 + xt$$

$$\therefore\ x = \frac{h_t - h_0}{h_0 \times t} = \frac{\cdot 63}{35 \times 100} = \frac{1}{5550} \text{ nearly.}$$

N.B. This is known as Dulong and Petit's method.

(39.) In another experiment of the same kind the height of the mercury column at 0° C. was 27·53 centimetres, and the difference of level of the two surfaces was 10·15 millimetres, while the temperature of the hot mercury was 200° C. Find from this experiment the mean coefficient of expansion of mercury between 0° C. and 200° C.

Answer. $\frac{1}{5425}$ nearly.

(40.) Determine the mean value of the coefficient of absolute expansion of mercury between 0° C. and 300° C. from an experiment in which the height of the cold column was 25·72 centimetres, and the difference of level of the surfaces of mercury in the two tubes was 14·56 millimetres, while the temperature of the hot tube was kept at 300° C.

Answer. $\frac{1}{5299}$ nearly.

(41.) In another experiment the heights of the two columns of mercury, as determined by a cathetometer, were 22·2 and 22·6 centimetres respectively, while their temperatures were 0° C. and 100° C. Find the mean coefficient of expansion. *Answer.* $\frac{1}{5550}$.

Note.—A serious objection to this method is that the difference of level of the mercury in the two tubes is necessarily very small, and even a small error in the observed value produces a considerable error in the result.

(42.) One of the tubes in an apparatus similar to that of Dulong and Petit contained a column of water 155 centi-

metres high, and the other tube contained a column of another liquid 317 centimetres high. At the temperature of 10° C. these two columns balanced each other.

Find (1) what was the density of this second liquid, that of water at 10° C. being ·99974, and (2) calculate its coefficient of expansion from the fact that when the temperature of its tube was raised to 25° C. the height of the column was raised to 318·1 centimetres, the temperature of the water remaining the same as before.

(1) Let x = density of the second liquid ; then by Example 38

$$317 \times x = 155 \times \cdot 99974$$

$$\therefore x = \frac{155 \times \cdot 99974}{317} = \cdot 4888 \text{ nearly.}$$

(2) Let y = mean coefficient of expansion of the second liquid for 1° C. ; then

$$1 + 15y = \frac{318 \cdot 1}{317} \therefore y = \frac{1 \cdot 1}{317 \times 15} = \frac{1}{4323} \text{ nearly.}$$

(43.) The density of mercury at 68° F., with respect to water at the same temperature, is found by experiment to be 13·568, and at 212° F. it is 13·932. If the expansion of mercury between these points be $\frac{1}{69}$ of its volume at the lower temperature, find that of water between the same points.

Let v_t represent the volume of a given mass of mercury at $t°$ F.

v'_t represent the volume of a given mass of water at $t°$ F.

and let x represent the expansion of unit volume of water between the two given temperatures.

Then, as in equation 1 of Example 38, we have

$$v_{212} = v_{68} \times \frac{70}{69}$$

$$\text{and } v'_{212} = v'_{68} \times (1 + x).$$

But by the conditions of the problem

$$\frac{v'_{68}}{v_{68}} = 13{\cdot}568, \text{ and } \frac{v'_{212}}{v_{212}} = 13{\cdot}932\ ;$$

$$\text{therefore } 1 + x = \frac{v'_{212}}{v'_{68}} = \frac{13{\cdot}932}{13{\cdot}568} \times \frac{70}{69} = 1{\cdot}0417$$

$$\therefore x = {\cdot}0417.$$

(44.) What fraction of its volume at 60° F. is the expansion of a substance for each additional degree of temperature if it expands the ·0006th part of the volume which it has at 32° F. for each degree above 32° F.?

Using a notation similar to that employed in the last example,

$$\frac{v_{60}}{v_{32}} = 1 + 28 \times {\cdot}0006 = 1{\cdot}0168,$$

$$\text{and } \frac{v_{60+t}}{v_{32}} = 1 + (28 + t) \times {\cdot}0006.$$

$$\text{Hence } \frac{\delta v}{v_{60}} = \frac{v_{60+t} - v_{60}}{v_{60}} = \frac{{\cdot}0006 \times t}{1{\cdot}0168} = {\cdot}00059 \times t$$

$$= {\cdot}00059 \text{ when } t = 1° \text{ F.}$$

RELATIVE EXPANSION.

(1.) The graduations of a glass measure are correct at 0° C., and at 15° C. it appears to contain 150 cubic centimetres of mercury. What quantity does it actually contain?

Let m = the number of grammes of mercury, then its volume at 15° C. is

$$\frac{m}{13{\cdot}6}\left(1 + \frac{15}{5550}\right) = \frac{m \times 5565}{13{\cdot}6 \times 5550} \text{ c.c.}$$

Again,

$$150 \text{ c.c. of glass at } 0° \text{ C. become } 150\left(1 + \frac{3 \times 15}{115607}\right) \text{ c.c.}$$

at 15° C.

$$\therefore \frac{m \times 5565}{13{\cdot}6 \times 5550} = 150 \times \frac{115652}{115607}.$$

Hence $m = 2035{\cdot}3$ grammes.

(2.) At the temperature of 23° C. a glass flask is exactly filled by 5,325 grammes of mercury. What would be its capacity at 0° C.?

Let v_t = capacity of flask in c.c.'s at t° C.
κ = coefficient of cubical expansion of the glass.

The volume of 5,325 grammes of mercury at 23° C. is

$$v_{23} = \frac{5325}{13 \cdot 596}\left(1 + \frac{23}{5550}\right) = 393 \cdot 28 \text{ c.c.}$$

and since the capacity of the flask at 0° C. is

$$v_0 = \frac{v_{23}}{1 + \kappa t} = v_{23} \times \frac{115607}{115676}, \text{ since } \kappa = \frac{3}{115607},$$

$$\therefore v_0 = 393 \cdot 28 \times \frac{115607}{115676} = 393 \cdot 048 \text{ c.c.'s.}$$

(3.) At the temperature of 14° C. a glass flask just holds 2,730 grammes of mercury. How much mercury will escape if the temperature be steadily raised to 100° C.?

$$\text{In this case } v_{14} = \frac{2730}{13 \cdot 596} \times \frac{5564}{5550} = 201 \cdot 3 \text{ c.c.'s.}$$

$$v_{100} = v_{14} \times \frac{115907}{115607} = 201 \cdot 823 \text{ c.c.'s.}$$

The mass of mercury which the flask can hold at 100° C. is therefore

$$201 \cdot 823 \times 13 \cdot 596 \times \frac{5550}{5650} = 2695 \cdot 4 \text{ grammes,}$$

and the quantity of mercury which escapes is therefore

$$2730 - 2695 \cdot 4 = 34 \cdot 6 \text{ grammes.}$$

(4.) A glass specific-gravity bottle can just contain 565 grammes of mercury at 0° C. The coefficient of linear expansion of the glass for 1° C. being $\frac{1}{118100}$, to what temperature must the bottle and its contents be raised so that 3 grammes of mercury may escape?

Let $x°$ C. = required temperature; then with the usual notation we have

$$v_x = \frac{565}{13{\cdot}596}\left(1 + \frac{3x}{116100}\right) \quad . \quad . \quad . \quad . \quad (1)$$

The mass of mercury remaining in the bottle at $x°$ C. is 562 grammes, and the volume of this at $x°$ C. is

$$v_x = \frac{562}{13{\cdot}596}\left(1 + \frac{x}{5550}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equations 1 and 2 we have

$$\frac{565}{13{\cdot}596}\left(1 + \frac{3x}{116100}\right) = \frac{562}{13{\cdot}596}\left(1 + \frac{x}{5550}\right)$$

$$\therefore x = \frac{3 \times 5550 \times 38700}{18613650} = 34{\cdot}62° \text{ C. nearly.}$$

(5.) A cylindrical glass tube 75 centimetres long, and of one centimetre internal diameter, is closed at one end and open at the other. At the temperature of the room, which is 15° C., it contains a column of mercury 74·5 centimetres long. By how much must the temperature rise so that the mercury may just fill the whole tube?

The quantity of mercury in the tube at $x°$ C.

$$= \frac{\pi}{4} \times 74{\cdot}5 \times 13{\cdot}596 \times \frac{5550}{5565} \text{ grammes}$$

$$= 793{\cdot}39 \text{ grammes.}$$

The volume of this mercury in the tube at $x°$ C.

$$= \frac{793{\cdot}39}{13{\cdot}596} \times \left(1 + \frac{x}{5550}\right) \text{ c.c.} \quad . \quad . \quad . \quad . \quad (1)$$

The interior capacity of the tube at $x°$ C. is

$$\frac{\pi}{4} \times 74{\cdot}5 \times \frac{115607 + x}{115652} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and since the mercury just fills the tube at $x°$ C. we have from (1) and (2)

$$\frac{115607 + x}{5550 + x} = \frac{115652}{5565}$$

$$\text{whence } x = \frac{266{\cdot}9}{19{\cdot}782} = 13{\cdot}49° \text{ C.}$$

(6.) A vessel is completely filled with mercury, and 57·5 grammes of iron are dropped in, the temperature of the room being 17° C. The density of iron at 0° C. being 7·2, and that of mercury 13·596, what quantity of mercury will be expelled from the vessel?

The volume of the iron at 17° C.

$$= \frac{57{\cdot}5}{7{\cdot}2}\left(1 + \frac{3 \times 17}{88889}\right) = 7{\cdot}99 \text{ c.c.}$$

But the volume of the mercury expelled is equal to that of the iron at 17° C., and therefore the quantity of mercury expelled is

$$7{\cdot}99 \times 13{\cdot}596 \times \frac{5550}{5567} = 108{\cdot}31 \text{ grammes.}$$

(7.) If the temperature of the room had been 25° C. what would have been the quantity of mercury displaced?

Answer. 108·18 grammes approximately.

(8.) If the quantity of iron had been 1 kilogramme, and the temperature of the room 20° C., what would have been the quantity of mercury expelled?

Answer. 1887·16 grammes approximately.

(9.) A glass flask at the temperature of melting ice contains exactly 220 grammes of water. It is then heated up to 80° C., and 5·833 grammes of water escape. The co-efficient of cubical expansion of the glass for 1° C. being $\frac{1}{38836}$, find the mean coefficient of expansion of water for 1° C. between 0° C. and 80° C.

Let x be the mean coefficient of expansion of water for

1° C. between 0° C. and 80° C., and let d_t be the density of water at t° C.

The capacity of the flask at 0° C. $= \dfrac{220}{d_0}$ c.c.

" " " $80^\circ \text{ C.} = \dfrac{220}{d_0} \times \left(1 + \dfrac{80}{38536}\right)$

$$= \frac{220{\cdot}46}{d_0} \text{ c.c.}$$

But $d_t = \dfrac{d_0}{1 + xt}$; therefore the water which just filled the flask at 0° C. has at 80° C. a volume $= \dfrac{220}{d_0}(1 + 80x)$; therefore the volume of water which escapes is

$$\frac{220}{d_0}(1 + 80x) - \frac{220{\cdot}46}{d_0}$$

$$= \frac{17600x - {\cdot}46}{d_0} \text{ c.c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But this quantity is 5·833 grammes, and its volume at 80° C. is

$$= \frac{5{\cdot}833}{d_{80}} = \frac{5{\cdot}833}{d_0}(1 + 80x) \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2) we have therefore

$$17600x - {\cdot}46 = 5{\cdot}833\,(1 + 80x),$$

whence $x = {\cdot}000367$ approximately.

(10.) A glass vessel with a perforated stopper contained a small piece of iron weighing 77 grammes, and was filled up at 0° C. with 135 grammes of mercury. The vessel was then placed in an oil bath and heated up to 115° C., and 2·458 grammes of mercury escaped. Calculate from the data of this experiment the coefficient of cubical expansion of iron.

Assuming that the density of iron at 0° C. is 7·78, and that of mercury 13·59, then

$$\left.\begin{array}{ll} \text{volume of the mercury at } 0^\circ \text{ C.} & = \dfrac{135}{13{\cdot}59} = 9{\cdot}9338 \text{ c.c.} \\ \text{,, \quad ,, \quad iron \quad ,,} & = \dfrac{77}{7{\cdot}78} = 9{\cdot}8972 \text{ c.c.} \end{array}\right\}$$

But the capacity of the glass vessel at any temperature must be equal to the sum of the volumes of the substances which fill it, and therefore

capacity of the flask at 0° C.

$$= 9{\cdot}9338 + 9{\cdot}8972 = 19{\cdot}831 \text{ c.c.}$$

capacity of the flask at 115° C.

$$= 19{\cdot}831\left(1 + \frac{115}{38536}\right) = 19{\cdot}890 \text{ c.c.} \quad . \quad . \quad . \quad (1)$$

Also the mass of the mercury in flask at 115° C.

$$= 135 - 2{\cdot}458 = 132{\cdot}542 \text{ grammes,}$$

and the volume of the mercury in flask at 115° C.

$$= \frac{132{\cdot}542}{13{\cdot}59}\left(1 + \frac{115}{5550}\right) = 9{\cdot}955 \text{ c.c.}$$

But the volume of the iron at 115° C. $= \dfrac{77}{7{\cdot}78}(1 + 115x)$ c.c.;

therefore the sum of the volumes of the mercury and iron which fill the flask at 115° C. is

$$9{\cdot}8972\,(1 + 115x) + 9{\cdot}955 \text{ cubic centimetres} \quad . \quad . \quad (2)$$

Equating the values from (1) and (2), we have

$$9{\cdot}8972\,(1 + 115x) + 9{\cdot}955 = 19{\cdot}890,$$

and from this equation we get

$$x = \tfrac{1}{30110}.$$

(11.) Another glass flask with a perforated stopper contained 100 grammes of iron, and when placed in melting ice was filled up with 120 grammes of mercury. The temperature was then raised to 100° C., and it was found that 1·974 gramme of mercury escaped. Taking the densities of iron and mercury the same as in the last example, find the coefficient of cubical expansion of this specimen of iron.

Answer. $\frac{1}{27941}$.

(12.) A specific gravity bottle with a perforated stopper was filled up at 0° C. with 138 grammes of mercury, and after being placed in an oil bath and heated to 150° C. it

was found that 3·13 grammes of mercury had escaped. Having given that the coefficient of absolute expansion of mercury is $\frac{1}{5550}$, find that of the glass.

Answer. ·0000213 nearly. (See also Example 34, p. 33.)

(13.) A glass flask is filled at 0° C. with 620 grammes of mercury. It is then heated to 100° C., and on being weighed again it is found to contain only 610·61 grammes of mercury. The density of mercury at 0° C. being 13·596, and its mean coefficient of expansion for 1° C. being $\frac{1}{5550}$, find the coefficient of expansion of the glass.

Answer. $\frac{1}{38459}$ approximately.

(14.) A glass bottle with a perforated stopper was filled with 124 grammes of water at 0° C., and after being heated to 100° C. it was found that 5·75 grammes of water had escaped. The coefficient of cubical expansion of this glass was known to be $\frac{1}{38460}$. Find from this experiment the expansion of unit volume of water between 0° C. and 100° C.

Answer. By the method of Example 9 we find that the expansion of unit volume is ·05136.

Note.—The student will notice the value of the determination, by Dulong and Petit's or Regnault's method, of the absolute expansion of mercury independently of that of the containing vessel; for, knowing the absolute expansion of mercury, we can find the absolute expansion of any glass vessel, and then by observing the apparent expansion of any liquid which fills that vessel we can determine the absolute cubical expansion of the liquid after correcting for the expansion of the vessel.

(15.) A glass vessel which had a narrow neck with a mark on it was filled up to this mark at 0° C. with 22·37 grammes of benzol. The vessel was then heated to 50° C., and all the liquid which passed above the mark was removed, and on weighing again it was found that the quantity of benzol remaining in the flask was 21·08 grammes. The co-

efficient of cubical expansion of the glass being $\frac{1}{38536}$, find the coefficient of expansion of the benzol.

Answer. ·00118 nearly.

(16.) A piece of glass tube was closed at one end and drawn out at the other to a capillary point. The quantity of water which just filled it at 0° C. was 172·5 grammes. The apparatus was then heated to 100° C., and on weighing again it was found that 8·346 grammes of water had been expelled. The coefficient of expansion of the glass being $\frac{1}{38536}$, calculate the expansion of unit volume of water between 0° C. and 100° C. *Answer.* ·05357.

(17.) At 0° C. a weight-thermometer was just filled by 251·5 grammes of mercury. On weighing the thermometer after its temperature had been raised to 100° C. it was found to contain only 247·691 grammes of mercury. The coefficient of cubical expansion of the glass being ·000026, calculate from the data of this experiment the coefficient of absolute expansion of the mercury. *Answer.* $\frac{1}{5550}$.

(18.) What would be the value of the coefficient of apparent expansion of mercury in this glass?

The apparent expansion is the difference between the actual expansion of the mercury and of the glass, and is $\therefore \frac{1}{6493}$ nearly.

(19.) An iron bottle contains a piece of platinum and is filled up at 0° C. with mercury. What must be the ratio of the quantities of mercury and platinum so that there may be no apparent expansion when the bottle is heated up to 100° C.?

The following 'constants' are to be used :—

Coefficient of cubical expansion of platinum for 1° C. $= \frac{1}{37700}$
" " " mercury " $= \frac{1}{5550}$
" " " iron " $= \frac{1}{28200}$

density of mercury at 0° C. $= 13·6$
" platinum " $= 21·0$.

Let m = required mass of the platinum in grammes
m' = " " mercury "

then the bulk of the platinum at $0°$ C. $= \dfrac{m}{21}$ c.c.

" " mercury " $= \dfrac{m'}{13{\cdot}6}$ c.c.

and therefore the capacity of the iron bottle $= \left(\dfrac{m}{21} + \dfrac{m'}{13{\cdot}6}\right)$ c.c.

When the temperature rises from $0°$ C. to $t°$ C. the sum of the volumes of the platinum and mercury will become

$$\frac{m}{21}\left(1 + \frac{t}{37700}\right) + \frac{m'}{13{\cdot}6}\left(1 + \frac{t}{5550}\right) \quad . \quad . \quad (1)$$

and the capacity of the iron bottle will be

$$\left(\frac{m}{21} + \frac{m'}{13{\cdot}6}\right) \times \left(1 + \frac{t}{28200}\right) \quad . \quad . \quad . \quad . \quad (2)$$

and if there is no ***apparent*** expansion these two expressions must be equal; therefore, equating (1) and (2), we get

$$\left(\frac{m}{21} + \frac{m'}{13{\cdot}6}\right) \times \frac{1}{28200} = \frac{m}{21 \times 37700} + \frac{m'}{13{\cdot}6 \times 5550}$$

$$\text{whence } \frac{m}{m'} = \frac{21 \times 2265 \times 377}{13{\cdot}6 \times 555 \times 95} = 25 \text{ approximately.}$$

THERMOMETERS.

(1.) A graduated glass tube with a cylindrical bore contains a thread of mercury which occupies 150 subdivisions at $0°$ C. What number of subdivisions will it occupy at $150°$ C?

Let x = required number of subdivisions
a = length of 1 subdivision at $0°$ C.
κ = coefficient of linear expansion of the glass;
then $a(1 + \kappa t)$ = length of a subdivision at $t°$ C.
and $xa(1 + \kappa t)$ = " " mercury thread at $t°$ C.;
and if r = radius of the bore at $0°$ C., then the capacity of x ubdivisions of the tube at $t°$ C. is $\pi r^2 xa(1 + 3\kappa t)$.

But this being the space occupied by the mercury at the temperature t° C. is $\pi r^2 \times 150a\left(1+\frac{t}{5550}\right)$, and $\therefore$ substituting the values for t and κ we have

$$x\left(1+\frac{3\times 150}{115607}\right)=150\left(1+\frac{150}{5550}\right)$$

$$\text{whence } x=\frac{150\times 5700\times 115607}{5550\times 116057}=153{\cdot}46 \text{ approx.}$$

The result may be arrived at by a much shorter method as follows :—

Let E_m be the expansion of unit volume of mercury for t° C.

E_g " " " " glass "

then if E_m and E_g were equal x would always be 150, but, as they are not equal,

$$\frac{x}{150}=\frac{E_m}{E_g}=\frac{1+\frac{150}{5550}}{1+\frac{3\times 160}{115607}}=\frac{5700\times 115607}{5550\times 116057}$$

$$\therefore x=153{\cdot}46 \text{ nearly.}$$

(2.) What number of subdivisions would the mercury occupy in this tube at 100° C.? *Answer.* 152·31 nearly.

(3.) A piece of empty thermometer tube weighed 14·51 grammes. A thread of mercury which occupied 43 millimetres of the bore was drawn up into the tube, and the whole weighed 14·5215 grammes. The temperature of the room being 12° C., find the diameter of the bore of this tube.

Let x be the diameter of the bore in centimetres

d be the density of mercury at 12° C.

The quantity of mercury which fills a length of 43 millimetres of the tube at 12° C. is $14{\cdot}5215-14{\cdot}51={\cdot}0115$ of a gramme.

$$\text{Then } \pi\times\left(\frac{x}{2}\right)^2\times 4{\cdot}3\times d={\cdot}0115$$

$$\therefore x^2=\frac{4}{\pi}\times\frac{{\cdot}0115\times}{4{\cdot}3}\ \frac{5562}{13{\cdot}596\times 5550}\quad . \quad . \quad . \quad (1)$$

$$\text{since } d = \frac{13{\cdot}596}{1+\frac{12}{5550}} = \frac{13{\cdot}596 \times 5550}{5562}$$

From (1) we get, by extracting the square root,

$$x = {\cdot}0158 \text{ centimetre nearly.}$$

(4.) Two grammes was the difference in weight of a glass tube when empty and when containing a thread of mercury which occupied a length of 15·5 centimetres. The temperature of the room being 15° C., find the diameter of the tube. *Answer.* ·11 centimetre nearly.

(5.) The bore of the tube of a Centigrade thermometer had a diameter of ·2 of a millimetre at 0° C., and the inside diameter of the spherical bulb was 9 millimetres. At 0° C. the mercury just filled the bulb up to the bottom of the tube. Assuming that the relative expansion of mercury in glass is $\frac{1}{6480}$, find the length of a Centigrade degree on this thermometer.

Let x = number of millimetres in one Centigrade degree; then the bulk of the mercury contained in this length of the tube at 0° C. is $\pi \times ({\cdot}1)^2 \times x$ cubic millimetres.

But the bulk of the mercury contained in the bulb at 0° C. is $\frac{4}{3}\pi \times (4{\cdot}5)^3$ cubic millimetres, and, since the volume of the mercury which occupies one subdivision of the tube is equal to the apparent expansion of the mercury in the bulb for 1° C., we have

$$\frac{4}{3}\pi \times (4{\cdot}5)^3 \times \frac{1}{6480} = \pi \times ({\cdot}1)^2 \times x$$

$$\text{From this we get } x = \frac{4 \times (4{\cdot}5)^3}{3 \times 6480 \times ({\cdot}1)^2}$$

$$= 1{\cdot}875 = 1{\cdot}9 \text{ millimetre nearly.}$$

(6.) Two mercurial thermometers were made of the same kind of glass. The diameter of the interior of the bulb of one of them was 7·5 millimetres, and that of the bore of its tube was ·25 of a millimetre. The diameter of the interior

of the bulb of the other thermometer was 6·2 millimetres, and that of the bore of its tube was ·15 of a millimetre. Compare the lengths of one Centigrade degree on the scales of these two thermometers.

$$\textit{Answer.}\quad \frac{\text{Length of a degree on second}}{\text{length of a degree on first}} = 1·569.$$

(7.) The capillary tube of a thermometer stem was divided into a series of equal parts whose capacity at 0° C. was ·01 of a cubic millimetre. The interior length of a cylindrical glass bulb which was to be fused to the end of this tube was 2·5 centimetres. What must have been the interior diameter of this cylindrical bulb so that each subdivision of the tube should correspond to $\frac{1}{10}$th of a Centigrade degree?

Let x be the radius of the interior of the bulb in millimetres; then

the bulk of the mercury which fills the bulb at 0° C.
$= \pi \times x^2 \times 25$ cubic millimetres

the bulk of the mercury which fills 10 subdivisions of the tube = ·1 cubic millimetre.

But this is the apparent expansion for 1° C. of the mercury which fills the bulb at 0° C. and

$$\therefore \frac{\pi x^2 \times 25}{6480} = ·1$$

whence $x = 2·872$ millimetres

$\therefore$ diameter required is 5·74 millimetres nearly.

(8.) A glass tube was carefully graduated into 250 divisions of equal capacity. The difference between the weights of the empty tube and of the tube when containing a thread of mercury which occupied 33 subdivisions of the tube was 5·32 centigrammes. What must be the interior diameter of a spherical glass bulb which would have to be attached to the end of the tube so that the 250 divisions should represent 200 Centigrade degrees?

Answer. 2·638 centimetres.

(9.) Suppose that we have to construct a mercury thermometer by means of which we can measure temperatures of

$+200^\circ$ C. and of -40° C. What must be the ratio of the capacity of the interior of the bulb to that of the rest of the tube?

Let $v =$ capacity of one subdivision of the tube.
$nv =$ „ the bulb.

Since the range is to be from -40° C. to $+200^\circ$ C., the relative expansion of the mercury in the bulb for a change of temperature of 240° C. must be $240\,v$, and

$$\therefore\ nv \times \frac{240}{6480} = 240\,v\,;$$

$$\therefore\ nv = 6480\,;$$

$$\therefore\ \frac{\text{capacity of bulb}}{\text{capacity of tube}} = \frac{6480}{240} = \frac{27}{1}.$$

(10.) A cylindrical glass tube of ·1 millimetre internal diameter was attached to a cylindrical glass bulb one centimetre long and one centimetre inside diameter. The tube was divided into millimetres, and at 0° C. the mercury which was introduced filled the bulb and a length of 3 millimetres of the tube. By what amount would the mercury rise in the tube for each increment of 1° C.?

Answer. 15·4 millimetres approximately.

(11.) An empty thermometer with a spherical bulb weighs 15 grammes. From its junction with the bulb the tube is graduated in millimetres, and is assumed to be of uniform cylindrical bore. Some mercury is introduced and at 0° C. just fills the bulb and 15 subdivisions of the tube. The weight of the tube and its contents is 16·53 grammes. Some more mercury is introduced, and when the instrument is placed in melting ice, the top of the mercury is at the 21st subdivision, and the weight of the whole is 16·583 grammes. What is the capacity of the bulb and that of each subdivision of the tube at 0° C.?

Let $m_1 =$ mass of the empty thermometer in grammes.
$m_2 =$ „ thermometer + first quantity of mercury.

Let m_3 = mass of the thermometer + final quantity of mercury
x = capacity of the bulb in cubic centimetres.
y = ,, each subdivision of the tube in c.c.
n = graduation corresponding to extremity of mer cury in first case.
n' = graduation corresponding to extremity of mercury in second case.

$$\left.\begin{array}{l}\text{Then } (x+ny)\ 13{\cdot}596 = m_2 - m_1 \\ \qquad (x+n'y)\ 13{\cdot}596 = m_3 - m_1\end{array}\right\}$$

$$\text{whence } y = \frac{m_3 - m_2}{13{\cdot}596\,(n'-n)} = \frac{16{\cdot}583 - 16{\cdot}53}{13{\cdot}596\,(21 - 15)} = \frac{{\cdot}053}{13{\cdot}596 \times 6}$$

$= {\cdot}0006497$ cub. centim. or ·65 cub. millim. nearly.

Substituting this value of y in the first equation, we get $x = {\cdot}10279$ c.c. nearly, or 102·79 cub. millim. nearly.

Note.—Problems are sometimes set in which it is stated that the first quantity of mercury introduced just fills the bulb to the zero of the scale. It is practically impossible to insure doing this. The above method of procedure gets over the difficulty.

(12.) A mercurial thermometer is partially immersed in a liquid whose temperature is required, the immersion being up to the 33rd degree of the scale, while the whole mercury column indicates 73° C. Find the true temperature of the liquid, taking the coefficient of the apparent expansion of mercury in glass at $\frac{1}{6480}$, and the temperature of the air at 15° C.

When the whole of the mercurial column is not immersed in the liquid under examination, but a portion of it is in a medium at a lower temperature, it is obvious that the indicated temperature of the liquid must be below the true temperature.

Let x = true temperature of the liquid in Centigrade degrees.
T = the temperature as indicated by the top of the mercury column.

Let v = the number of degrees of the mercury column exposed to the air.

θ = temperature of the air, which is assumed to be that of the exposed portion of the mercurial column.

If the thermometer indicated the actual temperature of the liquid, the temperature of the exposed part of the stem would be x and not θ, and at this temperature the length of the exposed portion would be

$$n \times \frac{1 + \dfrac{x}{6480}}{1 + \dfrac{\theta}{6480}}$$

and therefore the corrected length of the exposed portion would be greater than the actual length by a quantity

$$n\left\{\frac{1 + \dfrac{x}{6480}}{1 + \dfrac{\theta}{6480}} - 1\right\} = \frac{n(x - \theta)}{6480 + \theta};$$

and therefore the true temperature of the liquid is given by the equation

$$x = \mathrm{T} + \frac{n(x - \theta)}{6480 + \theta}.$$

Since θ is generally very small compared with 6480, it is usual to take the approximate formula

$$x = \mathrm{T} + \frac{n(x - \theta)}{6480}.$$

In the present case $\mathrm{T} = 73$, $n = 40$, and $\theta = 15°$ C. Substituting these values, we get

$$x = 73{\cdot}36° \text{ C. nearly.}$$

(13.) The bulb of a mercurial thermometer and the stem up to the zero of the scale are immersed in water at 100° C., while the remainder of the stem is exposed to the air

at the temperature 10° C. What will be the reading of this thermometer?

Using the approximate formula of the last example, we have

$$x = 100,\ n = \text{T},\ \theta = 10\ ;$$

$$\therefore\ 100 = \text{T} + \frac{\text{T} \times 90}{6480}\ ;$$

$$\therefore\ \text{T} = 98{\cdot}63^\circ\ \text{C}.$$

(14.) The bulb and the tube of a thermometer up to +22° C. are immersed in hot water, and the temperature as indicated by the thermometer is 67° C., while the temperature of the air is 20° C. What is the true temperature of the hot water? *Answer.* 67·33° C. nearly.

(15.) The temperature of the air in a laboratory was 13° C., and the indicated temperature of a hot liquid was 123° C., the tube of the thermometer being immersed up to the 10° mark. What was the true temperature of the liquid? *Answer.* 124·95° C.

(16.) What would have been the indicated temperature if the thermometer had been immersed in the liquid up to the 30° mark? *Answer.* 124·43° C.

(17.) There is half a cubic inch of mercury in a thermometer at 32° F., and when the temperature rises to 92° F. the mercury ascends 4 inches. What is the diameter of the bore of the glass tube? *Answer.* ·0286 inch.

BAROMETER CORRECTIONS.

(1.) On two separate occasions the height of the mercury column in a barometer was observed to be 74·8 centimetres but on the first occasion the temperature of the air was 16° C., while on the second occasion it was − 8° C. Reduce these two observations to what they would have been if the temperature had been 0° C.

If h_0, d_0, and h_t, d_t be the heights and densities of equivalent mercury columns at 0° C. and t° C. respectively, and remembering that the densities are inversely as the volumes, we have

$$\frac{h_0}{h_t} = \frac{d_t}{d_0} = \frac{1}{1 + kt} \left\{ \begin{array}{l} \text{where } k \text{ is the coefficient of cubical ex-} \\ \quad \text{pansion of mercury.} \end{array} \right.$$

$$\therefore\ h_0 = \frac{h_t}{1 + kt}.$$

Hence the first observation, when reduced to its equivalent at 0° C., becomes

$$h_0 = \frac{7 \cdot 48}{1 + \dfrac{16}{5550}} = 74 \cdot 585 \text{ centimetres.}$$

The second becomes

$$h_0' = \frac{74 \cdot 8}{1 - \dfrac{8}{5550}} = 74 \cdot 908 \text{ centimetres.}$$

(2.) What was the atmospheric pressure per square centimetre of surface on each of these occasions?

The pressure on a square centimetre is equal to the weight of a column of mercury of one square centimetre section, whose height and density are that of the barometric column.

$\therefore$ Pressure on first occasion $= 74 \cdot 585 \times 13 \cdot 596 = 1014 \cdot 058$ grammes weight.

Pressure on second occasion $= 74 \cdot 908 \times 13 \cdot 596 = 1018 \cdot 449$ grammes weight.

(3.) The heights of the barometer were observed simultaneously at two places, and found to be 76·2 and 72·8 centimetres, the temperature at the first place being 28° C., and at the second place 2° C. What would have been the equivalent values at 0° C.?

Answer. 75·817 and 72·774 centimetres.

(4.) Assuming that the extreme height of the barometric column in England is 80 centimetres, and that the extreme temperature is 25° C., what is the greatest possible value of the requisite correction to reduce the observation to its equivalent at 0° C.?

Answer. —3·6 millimetres approximately.

(5.) The height of the barometric column was observed to be 76·2 centimetres when the thermometer stood at 15° C. What correction would have to be applied to reduce this to the standard temperature of 0° C.?

Answer. —2·06 millimetres.

(6.) At the temperature of 18° R. the height of the barometric column was observed to be 73·3 centimetres. Reduce this to the standard temperature of 0° C.

Answer. 73·004 centimetres.

(7.) Reduce an observed barometric height of 76·44 centimetres at 20° C. to its equivalent at the standard temperature of 0° C. *Answer.* 76·165 centimetres.

(8.) An English barometer with a brass scale which was correctly graduated at 62° F. reads 29·71 inches when the temperature of the air is 45° F. Reduce this to the equivalent reading at 32° F.

Let a = coefficient of linear expansion of the scale for 1° F.
k = " cubical " mercury "
t = temperature of the air at the time of the observation.
t' = " " when the scale was graduated.

Since the temperature of the scale when this observation is made is lower than what it was when the scale was graduated, each apparent inch is less than a true inch by the quantity $a(t' - t)$.

$\therefore$ h_t apparent inches $= h_t(1 + a\overline{t - t'})$ true inches;
and $\therefore$ by the method of Example 1,

$$h_0 = h_t \frac{1 + a(t - t')}{1 + kt}$$

But

$$a = \frac{5}{9 \times 53333} = \cdot0000104\ ;\ k = \frac{5}{9 \times 5550} = \cdot0001001\ ;$$

$$h_t = 29\cdot71\ ;\ t = 45 - 32 = 13\ ;\ t' = 62 - 32 = 30\ ;$$

$$\therefore\ h_0 = 29\cdot71\ \frac{1 - 17 \times \cdot0000104}{1 + 13 \times \cdot0001001} = 29\cdot71 \times \frac{\cdot9998232}{1\cdot001301}$$

$= 29\cdot67$ inches nearly.

(9.) The same barometer on another occasion reads 30 inches at 60° F. What is the atmospheric pressure in true inches of mercury at 32° F.? *Answer.* 29·915 inches.

(10.) On another occasion this barometer indicated 30·09 inches at 58° F. Reduce this to true inches of mercury at 32° F. *Answer.* 30·01 inches.

(11.) The brass scale of a certain barometer was correctly graduated at 10° C. At what temperature will the observed reading require no temperature correction?

Let x° C. be the required temperature, and let y be the height of the mercury column as indicated by the scale at this temperature. Then, by the method adopted in Example 8, the length of the mercury column at x° C., reduced to its equivalent at 0° C., will be

$$\frac{y\left\{1 + \dfrac{x - 10}{53333}\right\}}{1 + \dfrac{x}{5550}}$$ and this $= y$ since no correction is required.

$$\therefore\ \frac{x - 10}{53333} = \frac{x}{5550}\ ;$$

whence $x = -1\cdot16$° C.

(12.) A barometer with a brass scale, which was correctly graduated at 0° C., stands at 77·8 centimetres on a certain occasion when the temperature is 20° C. What pressure in kilogrammes per square centimetre does this indicate?

Answer. 1·054 kilogrammes per square centimetre.

BOYLE'S LAW.

(1.) When the barometer stands at 76 centimetres, and the thermometer at 0° C., the space occupied by 1·293 grammes of dry air is one cubic decimetre (one litre). What space will this quantity of air occupy when the barometer stands at 52 centimetres, the temperature remaining constant?

According to Boyle's law the pressure of a given quantity of gas at a constant temperature varies inversely as the space it occupies. Hence, if p_1, v_1, p_2, v_2 represent corresponding values of the pressure and volume of a quantity of gas whose temperature is constant, Boyle's law asserts that

$$p_1 v_1 = p_2 v_2$$

$$\therefore v_2 = \frac{p_1}{p_2} \times v_1.$$

But in the present case $p_1 = 76$, $p_2 = 52$, $v_1 = 1000$ c.c.

$$\therefore v_2 = \frac{76 \times 1000}{52} = 1461 \cdot 5 \text{ c.c.}$$

(2.) A balloon containing 1,200 cubic metres of gas when the pressure corresponds to a mercury column of 77 centimetres, ascends until the barometer stands at 53 centimetres. What volume will the gas in the balloon now occupy, supposing that none has escaped?

Answer. 1743·4 cubic metres.

(3.) An air-bubble at the bottom of a pond 3 metres deep has a volume of one cubic millimetre. What space will it occupy when it just reaches the surface, the barometer standing at 76 centimetres and mercury being 13·6 times as heavy as the water in the pond?

The pressure at the bottom of the pond when expressed in terms of the equivalent mercury column is

$$p_1 = 76 + \frac{300}{13{\cdot}6} = \frac{1109{\cdot}6}{13{\cdot}6} \text{ centimetres.}$$

Pressure at the top is $p_2 = 76$ centimetres

$$\therefore\ v_2 = \frac{p_1}{p_2} \times v_1 = \frac{1109{\cdot}6}{13{\cdot}6} \times \frac{1}{76} = 1{\cdot}0735 \text{ cub. millimetres.}$$

(4.) One of Coxwell's balloons was filled with 90,000 cubic feet of coal gas, when the atmospheric pressure was equivalent to 30 inches of mercury. It ascended until the atmospheric pressure was equal to that of 7 inches of mercury. If the temperature had remained constant, what quantity of gas must have escaped by the valve?

By Boyle's law

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$\therefore\ \frac{v_2 - v_1}{v_1} = \frac{p_1 - p_2}{p_2} = \frac{23}{7}\ \therefore\ v_2 - v_1 = \frac{23}{7} \times v_1.$$

But the density of the expanded gas is only $\frac{7}{30}$ of its original density, and therefore the quantity of gas which escaped when reduced to its original density was

$$\frac{7}{30} \times \frac{23}{7} \times v_1 = \frac{23}{30} \times 90000 = 69000 \text{ cubic feet.}$$

(5.) The shorter branch of a Marriotte's tube (the well-known apparatus for testing Boyle's law) contains initially 5 cubic centimetres of air at the pressure of 76 centimetres of mercury, and the bore of the tube is such that a length of one centimetre has a capacity of one cubic centimetre. What will be the volume and pressure of the enclosed mass of air if 76 cubic centimetres of mercury are poured into the longer branch?

Let x be the height, in centimetres, through which the mercury rises in the shorter arm.

The difference of level of the surfaces of the mercury in the two branches is $\therefore$ $(76-2x)$ centimetres.

The space occupied by the air in the shorter arm is $(5-x)$ c.c., and the pressure upon it is $76+(76-2x)=2(76-x)$ centimetres of mercury.

Hence applying the equation

$$\frac{v_2}{v_1}=\frac{p_1}{p_2}$$

we have $\dfrac{5-x}{5}=\dfrac{76}{2(76-x)}$ whence $x=\dfrac{81\pm76\cdot164}{2}$

and since the smaller value of x is obviously the only admissible one, we get

$$x=2\cdot418 \text{ c.c.}$$

and $\therefore$ space occupied by the air is

$$5-x=5-2\cdot418=2\cdot582 \text{ c.c.}$$

Also the new pressure is $2(76-x)=2(73\cdot582)=147\cdot164$ centimetres.

(6.) The two branches of a Marriotte's tube are graduated in centimetres, and the sectional area of the bore is one square centimetre. There are 10 centimetres of air in the shorter branch, and the difference of level of the mercury in the two branches is 108 centimetres. The laboratory barometer stands at 75 centimetres. What is the pressure of the air in the tube, and what length would it occupy if it were under the atmospheric pressure only?

Answer. Pressure = 183 centimetres of mercury.
Volume = 24·4 c.c.

(7.) In another experiment with the same tube the air in the closed branch occupied 8 cubic centimetres when the mercury was at the same level in both branches, and the barometer stood at 76 centimetres. How much mercury had to be poured into the longer branch so as to compress the air to 3 cubic centimetres?

Let x be the number of cubic centimetres required,

then since the surface of the mercury in the closed branch ascends 5 centimetres, the difference of level of the mercury in the two branches is $(x-10)$ centimetres. Hence by Boyle's law

$$\frac{8}{3}=\frac{76+x-10}{76}$$

whence $x = 136\frac{2}{3}$ cubic centimetres.

(8.) A narrow glass tube of uniform cylindrical bore and closed at one end is supported vertically, with its open end upwards. It contains a column of dry air which occupies a length of 16 centimetres, which is surmounted by a column of mercury 4 centimetres long. The laboratory barometer standing at 76 centimetres and the temperature remaining constant, if the tube be inverted, what length will the air occupy?

Let v be the volume in cubic centimetres of a length of one centimetre of the bore of the tube, and let x be the length in centimetres which is occupied by the air in the second case.

Then $$\frac{v_2}{v_1}=\frac{x\times v}{16\times v}=\frac{x}{16}$$

Also the pressure of the air in first position $= 76+4 = 80$ centimetres.

The pressure of the air in second position $= 76 - 4 = 72$ centimetres.

And by Boyle's law

$$\frac{80}{72}=\frac{v_2}{v_1}=\frac{x}{16} \quad \therefore\ x = 17\tfrac{7}{9} \text{ centimetres.}$$

(9.) What would have been the length occupied by the air in the second position if the initial length of the air column had been 14 centimetres, that of the mercury column 10 centimetres, and the barometric pressure 74 centimetres? *Answer.* $18\frac{3}{8}$ centimetres.

(10.) A tube similar to the last contained some dry air which was separated from the external air by a column of mercury 8 centimetres long. When the tube was horizontal the air occupied a length of 12 centimetres, and when the tube was vertical with the open end upwards the air column was 10·8 centimetres long. The temperature being constant, what was the barometric pressure?

Let x be the atmospheric pressure in centimetres of mercury, then the pressure on the enclosed air when the tube is horizontal $= x$
the pressure on the enclosed air when the tube is vertical $= x + 8$.

$\therefore$ by Boyle's law

$$\frac{x+8}{x} = \frac{12}{10{\cdot}8} \quad \therefore x = 72 \text{ centimetres.}$$

(11.) If an error of one millimetre in excess were made in reading off the length of the air column in both positions, what would have been the indicated barometric height?

Answer. $72\frac{2}{3}$ centimetres.

(12.) A barometer tube one metre long is inverted to the depth of one centimetre in a deep trough of mercury, and contains a certain quantity of dry air which, at the beginning of the experiment, is at a pressure of 32 centimetres of mercury. The tube is then lowered into the trough until the pressure is equivalent to 44·2 centimetres of mercury. The barometric pressure being 75 centimetres, what length of the tube is occupied by the air in the second position?

Since the lower end of the tube is initially at one centimetre below the surface of the mercury in the trough, the length of the mercury column in the tube is

$$75 - 32 = 43 \text{ centimetres,}$$

and the length occupied by the air in the first position is

$$\therefore 99 - 43 = 56 \text{ centimetres.}$$

If x be the length occupied by the air in the second position, Boyle's law gives us

$$\frac{x}{56} = \frac{32}{44 \cdot 2} \therefore x = 40 \cdot 54 \text{ centimetres nearly.}$$

(13.) In another experiment with a similar apparatus the air in the tube occupied initially a length of 26 centimetres, and the surface of the mercury in the tube was 30 centimetres above that of the mercury in the trough. On raising the tube 20 centimetres the length of the air space was observed to be 42·8 centimetres, and the height of the mercury above that in the trough was 47·2 centimetres. What was the barometric pressure?

Let x be the barometric height in centimetres, then by Boyle's law

$$\frac{x - 47 \cdot 2}{x - 30} = \frac{26}{42 \cdot 8} \therefore x = 73 \cdot 8 \text{ nearly.}$$

(14.) An imperfectly exhausted barometer of uniform bore registers 75·1 centimetres when the laboratory barometer registers 76·2 centimetres, and the space above the mercury is 6 centimetres. What will be the true reading when this barometer indicates 74 centimetres?

Let x be the true barometric pressure in second case, then Pressure of the enclosed air in second case $= x - 74$ centimetres.

Pressure of the enclosed air in first case $= 76 \cdot 2 - 75 \cdot 1 = 1 \cdot 1$ centimetres, and by Boyle's law

$$\frac{x - 74}{1 \cdot 1} = \frac{6}{7 \cdot 1} \therefore x = 74 \cdot 93 \text{ centimetres nearly.}$$

(15.) The height of the top of a uniform barometer tube above the surface of the mercury in the tank is 83 centimetres. On account of an imperfect vacuum the barometer registers 71·5 centimetres when the barometric height is 72·5 centimetres. What will be the true barometric height when this barometer registers 74·8 centimetres?

Answer. 76·2 centimetres.

(16.) The air in the manometer (pressure gauge) attached to a condensing air-pump initially occupies 160 subdivisions of the tube and the pressure is 76 centimetres of mercury, which is also that of the external air. After a certain number of strokes of the piston, the air in the manometer is compressed to 20 subdivisions and the mercury has risen through a height of 50 centimetres. In what ratio has the quantity of air in the receiver been augmented if the temperature has remained constant?

The pressure of the air in manometer

$$= \frac{160}{20} \times 76 = 608 \text{ centimetres,}$$

and since the pressure of the air in the receiver balances this pressure and that of 50 centimetres of mercury, we have

$$p_2 = 608 + 50 = 658 \text{ centimetres.}$$

$$\text{Also } p_1 = 76.$$

$$v_1 = 1.$$

And when the temperature and the volume are constant the pressure must vary directly with the quantity of air in a given space;

$$\therefore \frac{\text{air in receiver finally}}{\text{air in receiver initially}} = \frac{p_2}{p_1} = \frac{658}{76} = 8 \cdot 658.$$

(17.) A barometer tube of uniform bore was held with its closed end downwards, and was filled with mercury up to a distance of 12 centimetres from the top, the remainder containing dry air. The tube was then closed with the finger and inverted in a deep trough of mercury and supported in a vertical position, so that the level of the mercury in the tube was 25·6 centimetres above that of the mercury in the trough, while the length of tube occupied by the air was 18·2 centimetres. What was the barometric pressure?

Let x be the barometric height in centimetres. When the tube is inverted in the mercury trough the pressure

of the air inside is obtained by Boyle's law from the equation,

$$p = \frac{12}{18 \cdot 2} \times x \quad . \quad . \quad . \quad . \quad (1).$$

But since the pressure of the air in the tube, together with the weight of a column of mercury 25·6 centimetres high, balances the atmospheric pressure x, we have also

$$p = x - 25 \cdot 6 \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we get $x = 75 \cdot 15$ centimetres nearly.

(18.) A closed barometer tube, the lower portion of which is immersed vertically in a deep trough of mercury, contains 2·5 c.c. of dry air, and the level of the mercury in the tube above that in the trough is 62·3 centimetres. The tube is then raised until the enclosed air occupies 3·5 c.c., and it is then found that the level of the mercury in the tube above that in the trough is 65·9 centimetres. Find from these data the barometric pressure.

Answer. 74·9 centimetres.

(19.) A glass vessel which can be closed by a stopcock is filled with dry air at the pressure of 75 centimetres of mercury, and is then connected with the upper part of a cistern barometer which has also a stopcock at the top. The bore of this tube is uniform and one square centimetre in section, and its top is 83 centimetres above the level of the mercury in the cistern. On opening both stopcocks the mercury in the barometer tube sinks and comes to rest, with its surface 9·3 centimetres above that of the mercury in the cistern. Find the capacity of this glass vessel.

Let v be the capacity of the glass vessel in cubic centimetres. On opening the stopcocks the air, which originally occupied a space of v c.c., now occupies a space of $v + 73 \cdot 7$ cubic centimetres.

Also the pressure of the air at first is 75 centimetres, and

after the stopcocks are opened it is $75-9·3 = 65·7$ centimetres. Applying Boyle's law we have therefore

$$\frac{v}{v+73·7} = \frac{65·7}{75}$$

$$\therefore v = 520·7 \text{ c.c. nearly.}$$

(20.) Another glass vessel with a stopcock was filled with dry air when the barometer stood at 77 centimetres, and was then connected with the upper part of a very wide barometer tube, whose length above the surface of the mercury in the cistern was 90 centimetres, and whose sectional area was 20 square centimetres. On opening the stopcocks the mercury in this tube descended till its surface was 40 centimetres above that of the mercury in the cistern. What was the capacity of the glass vessel?

Answer. 925 c.c.

(21.) Five cubic centimetres of dry air at atmospheric pressure are introduced into the vacuum of a barometer which previously stood at 76·2 centimetres, and occupy a volume of 8 cubic centimetres. By how much is the barometric column depressed?

Answer. 47·6 centimetres nearly.

(22.) A barometer tube of uniform cylindrical bore is lowered mouth downwards into a deep trough of mercury. The upper part of the tube contains dry air which occupies a length of 19 centimetres, and the surface of the mercury in the tube is 6 centimetres above that of the mercury in the trough. If the temperature remain constant and the barometric pressure be 75·2 centimetres, what will be the height of the mercury in the tube and the length of the air column if the tube be raised through a vertical height of 20 centimetres?

Let the required height of the mercury column $= x$ centimetres, then the length of the corresponding air column $= 25 + 20 - x = 45 - x$ centimetres, and if h be the height of the barometric column the pressure of the enclosed air in

the first position is $h - 6$ and in the second position it is $h-x$; hence, applying Boyle's law, we have

$$\frac{19}{45-x} = \frac{h-x}{h-6} = \frac{75{\cdot}2-x}{69{\cdot}2}.$$

Solving this quadratic equation we obtain

$$x = 60{\cdot}1 \pm 39{\cdot}28,$$

and it is obvious that the smaller value of x is the only one which satisfies the conditions of this problem. Hence

$$x = 60{\cdot}1 - 39{\cdot}28 = 20{\cdot}82 \text{ centimetres},$$

and corresponding length of air column $= 45 - x = 24{\cdot}18$ centimetres.

LAW OF CHARLES.

(1.) A very long horizontal glass tube of uniform bore is closed at one end and subdivided into parts of equal capacity. It contains a quantity of dry air which, at 0° C. occupies 273 subdivisions of the tube, and which is cut off from the external air by a small pellet of mercury. What numbe of subdivisions will the air occupy if its temperature be raised to 60° C.?

Charles, a Professor of Physics at Paris (born 1746, died 1823), discovered that when a quantity of gas is heated under constant pressure from 0° C. to 100° C. its volume increases by $\frac{1}{273}$rd of itself for each degree Centigrade, whatever be the nature of the gas.

Hence if v_t and v_0 be the volumes of the same quantity of gas at $t°$ C. and 0° C. respectively, applying Charles's law we have

$$v_t = v_0 \left(1 + \frac{t}{273}\right).$$

In the present case $v_0 = 273$, $t = 60$.

$$\therefore\ v_{60} = 273 \left(1 + \frac{60}{273}\right) = 333 \text{ subdivisions.}$$

(2.) If the tube and the air contained in it were cooled down to $-60°$ C., what space would the air occupy?

For the lowest temperature hitherto attainable Charles's law is found to hold good, and therefore we have

$$v_{60} = v_0 \left(1 - \frac{60}{273}\right) = 273 - 60 = 213 \text{ subdivisions.}$$

(3.) Assuming that Charles's law still held for such a low temperature as $-273°$ C. and that the air were still a gas at that temperature, what volume would it occupy?

Answer. $v_{273} = 273 \left\{1 - \frac{273}{273}\right\} = 0$

Note.—According to the dynamical theory of heat the temperature of a body depends upon the kinetic energy of its molecules, and in the case of a gas it maintains its volume in opposition to the external pressure by means of its molecular energy. If the gas were unable to maintain any volume at all we must suppose that it has lost all its molecular energy, and therefore that its temperature is absolutely zero. This imaginary temperature of $-273°$ C. is called the *absolute zero* of the air thermometer, and temperatures reckoned from this point are called absolute temperatures.

(4.) The coefficient of expansion of gases for 1° C. is $\frac{1}{273}$. What is the coefficient of expansion for 1° R. and for 1° F.?

Answer. For 1° R. $x = \frac{1}{218 \cdot 4}$

For 1° F. $x = \frac{1}{491 \cdot 4}$

(5.) Express all these coefficients in decimals to six places. *Answers.* ·003663; ·004579; ·002035.

(6.) What is the absolute temperature which corresponds to 0° C. and also to 0° F.?

To obtain the absolute temperature on the Centigrade scale we must add 273° C. to the measure of the temperature in that scale.

Hence $\quad T_1 = 0 + 273 = 273$

$$T_2 = -17\tfrac{7}{9} + 273 = 255\tfrac{2}{9}$$

(7.) A certain quantity of gas occupies 15 litres at 20° C. If the pressure remain constant, what space will this gas occupy at 80° C.?

Let v_1, t_1, v_2, t_2 be the corresponding values of the volume and temperature of a given quantity of gas when the temperature is constant, then by Charles's law

$$v_1 = v_0\left(1 + \frac{t_1}{273}\right) = v_0\,\frac{t_1 + 273}{273} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$v_2 = v_0\left(1 + \frac{t_2}{273}\right) = v_0\,\frac{t_2 + 273}{273} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$\therefore \dfrac{v_2}{v_1} = \dfrac{273 + t_2}{273 + t_1} = \dfrac{T_2}{T_1}$ where T is the absolute temperature.

In the present case

$$\frac{v_2}{15} = \frac{273 + 80}{273 + 20} = \frac{353}{293} \;\therefore\; v_2 = 18{\cdot}072 \text{ litres nearly.}$$

(8.) A quantity of gas occupies 30 cubic inches at the normal pressure and 62° F. What will be its volume at the temperature of freezing water?

Answer. 28·274 cubic inches.

(9.) A quantity of oxygen occupies 150 cubic centimetres at 15° C. What space will it occupy at the same pressure if the temperature be reduced to 0° C.?

Answer. 142·19 c.c.

(10.) A gas has its temperature raised from 8° C. to 72° C. and at the latter temperature it occupies 12 litres. The pressure being constant what was its original volume?

Answer. 9·7739 litres.

(11.) At 76 centimetres of barometric pressure and 0° C. the space occupied by 1·293 grammes of air is one litre. At what temperature will the mass of one litre of air be one gramme, the pressure remaining constant?

Let x° C. be the required temperature, then by Charles's law we have

$$\frac{273+x}{273} = \frac{1 \cdot 293}{1} \therefore x = 80^\circ \text{ C. nearly.}$$

Note.—The mass of a litre of air at 0° C. and 76 centimetres pressure varies with the intensity of gravity. But no correction on this account is necessary, as in weighing a litre of air the maximum error which could arise from this cause cannot exceed one milligramme, which is less than the possible error arising from any uncertainty in the determination of the quantity of aqueous vapour in the air.

(12.) At what temperature has carbonic acid gas the same density as oxygen gas at 0° C.? The mass of a litre of carbonic acid gas at the normal temperature and pressure is 1·977 grammes, and that of a litre of oxygen 1·430 grammes.

Answer. 104·4° C.

(13.) A litre of hydrogen at 0° C. and 76 centimetres pressure contains ·0896 gramme. At what temperature will the density of air be equal to that of hydrogen at 0° C., the pressure remaining constant? *Answer.* 3666·6° C.

(14.) At what temperature has air the same density as oxygen at 0° C.? *Answer.* −26·12° C.

(15.) How many grammes of air are there in a room which is $16 \times 18 \times 12$ feet, when the barometer stands at 76 centimetres, the thermometer at 25° C., and the aqueous vapour is neglected?

Since one cubic foot = 28316 c.c., the capacity of the room = $28316 \times 16 \times 18 \times 12$ c.c.

But at 25° C. and 76 centimetres pressure the mass of one cubic centimetre of air is $\cdot 001293 \times \frac{273}{298}$ gramme.

∴ quantity of air in the room

$$= 28316 \times 16 \times 18 \times 12 \times \cdot 001293 \times \frac{273}{298}$$

$$= 115918 \text{ grammes.}$$

$$= 115 \cdot 918 \text{ kilogrammes.}$$

(16.) What would have been the quantity of air in the room if the temperature had been 0° C., the pressure remaining the same? *Answer.* 126·533 kilogrammes.

(17.) A glass flask with a stopcock is filled with dry air at 0° C. and 76 centimetres pressure and then weighed. It is then opened, heated to 100° C., closed and weighed again, when it is found that 1·23 grammes of air have escaped. What was the capacity of this flask at 0° C.?

With the usual notation we find that

Capacity of flask at 100° C. $= v_0\left(1 + \frac{100}{38536}\right) = v.$

And v_0 c.c.s of air at 0° C. occupy at 100° C. a volume

$$= v_0\left(1 + \frac{100}{273}\right) = v',$$

$\therefore$ the volume of the air at 100° C. which escapes is

$$v' - v = v_0\left\{\frac{100}{273} - \frac{100}{38536}\right\} = v_0 \times \cdot 3637.$$

But the density of the air at 100° C. $= \cdot 001293 \times \frac{273}{373}$,

$\therefore$ the mass of the air which escapes

$$= v_0 \times \cdot 3637 \times \cdot 001293 \times \frac{273}{373} \text{ grammes,}$$

$$= 1\cdot 23 \text{ grammes by the question;}$$

$$\therefore v_0 = \frac{1\cdot 23 \times 373}{\cdot 3637 \times 273 \times \cdot 001293} = 3573\cdot 6 \text{ c.c.}$$

(18.) Another glass flask was filled with dry air at 10° C. and 76 centimetres barometric pressure, and was then weighed. It was then heated to 100° C., and the stopcock closed. Upon weighing it again it was found that ·83 gramme of air had escaped. What was the interior capacity of this flask at the lower temperature?

Answer. 2778·3 c.c.

(19.) This experiment was repeated with another flask, and on weighing it the second time it was found that ·92 of a gramme of air had escaped. What was the capacity of the flask at 0° C.? *Answer.* 2673 c.c. nearly.

GENERAL MEASUREMENTS OF GASES.

(1.) Three litres of dry gas are measured off at 15° C. and 76·7 centimetres barometric pressure. Find the volume of this gas at the standard temperature of 0° C., and the standard barometric pressure of 76 centimetres of mercury.

The laws of Boyle and Charles may be combined into the statement that 'the product of the volume and pressure of any gas is proportional to the absolute temperature.'

Hence, if v, p, T be the actual volume, pressure, and absolute temperature, and v_0 the volume when reduced to the standard pressure p_0, and the standard absolute temperature T_0, we shall have

$$\frac{vp}{T} = \frac{v_0 p_0}{T_0},$$

$$\therefore v_0 = v \times \frac{p}{p_0} \times \frac{T_0}{T}.$$

In the present case $\left.\begin{array}{l} p = 76{\cdot}7 \\ p_0 = 76 \end{array}\right\}$ $\left.\begin{array}{l} T = 288 \\ T_0 = 273 \end{array}\right\}$ $v = 3$,

$$\therefore v_0 = 3 \times \frac{76{\cdot}7}{76} \times \frac{273}{288} = 2{\cdot}8699 \text{ litres.}$$

(2.) A quantity of gas measures 30 cubic centimetres at 20° C. and 36 centimetres barometric pressure. What space will it occupy at the standard temperature and pressure?

Answer. 13·24 c.c. nearly.

(3.) Reduce 2·5 litres of dry air at 25° C. and under a pressure of 74 centimetres of mercury, to the corresponding volume at standard temperature and pressure.

Answer. 2·23 litres.

(4.) A quantity of dry air at 15° C. exerts a pressure of 16 lbs. to the square inch on the sides of the containing vessel, which are assumed to be inextensible. What will be the pressure per square inch if the temperature be raised to 100° C.?

By the combination of the laws of Boyle and Charles, as stated in the solution of Example 1, we see that

$$\frac{\mathrm{V\,P}}{\mathrm{T}} = \text{a constant};$$

and then if v remain constant, and P_1, T_1; P_2, T_2 be corresponding values of the pressure and absolute temperature,

$$\frac{\mathrm{P}_2}{\mathrm{P}_1} = \frac{\mathrm{T}_2}{\mathrm{T}_1} \therefore \mathrm{P}_2 = \mathrm{P}_1 \times \frac{\mathrm{T}_2}{\mathrm{T}_1}.$$

$$\text{But } \left.\begin{array}{l} \mathrm{T}_2 = 273 + 100 = 373 \\ \mathrm{T}_1 = 273 + 15 = 288 \end{array}\right\} \mathrm{P}_1 = 16;$$

$$\therefore \mathrm{P}_2 = 16 \times \frac{373}{288} = 20{\cdot}72 \text{ lbs. approximately.}$$

(5.) If the space occupied by the air in an air thermometer be kept constant while its pressure changes from 76 to 102 centimetres of mercury, what is the change of temperature, the initial temperature being 10° C.?

Answer. 96·8° C. nearly.

(6.) Two vessels contain air at the same pressure, namely, 76 centimetres of mercury, but at different temperatures, 50° C. and 60° C. If the temperature of each be increased by 10° C., find which has its pressure the most increased?

Answer. The one whose temperature was originally lowest.

(7.) Two inextensible vessels contain air at the same temperature, namely, 14° C., but at the respective pressures of 76·2 and 76·5 centimetres of mercury. The temperature of each being increased by 20° C., find which has its pressure most increased.

Answer. The one whose original pressure was 76·5 centimetres.

(8.) If 3 litres and 5 litres of two different gases, at the same temperature of 13° C., but at the respective pressures of 74·2 and 76·3 centimetres, be mixed together, the volume of the mixture being 6 litres and its temperature 8° C., determine the pressure.

Let v_1, p_1; v_2, p_2; v, p, the volumes and pressures of the two gases and of their mixture all at the same initial temperature, then we have

$$\begin{aligned} v\,p &= v_1\,p_1 + v_2\,p_2 \\ &= 3 \times 74{\cdot}2 + 5 \times 76{\cdot}3 = 604{\cdot}1. \end{aligned}$$

But by the question $v = 6$,

$$\therefore\ p = \frac{604{\cdot}1}{6} \text{ centimetres.}$$

In the next place let the temperature of the mixture change from 13° C. to 8° C., the volume remaining constant, and let P′ be the final pressure, then

$$\frac{p'}{p} = \frac{8 + 273}{13 + 273} = \frac{281}{286}$$

$$\therefore\ p' = \frac{281}{286} \times \frac{604{\cdot}1}{6} = 98{\cdot}923 \text{ centimetres.}$$

(9.) Two cubic feet of oxygen, 5 cubic feet of nitrogen, and one cubic foot of hydrogen, all at the same temperature, and at a pressure of 76 centimetres of mercury, were mixed together, and the mixture was compressed into a space of 4 cubic feet. When the temperature was reduced to its original value what was the pressure?

Answer. 152 centimetres.

(10.) The air in an extensible spherical envelope has its temperature gradually raised from 0° C. to 20° C., and the envelope is allowed to expand until its radius is four times its original length. Compare the pressures of the air in the two cases. *Answer.* $p_1 = p_2 \times 59{\cdot}6$ nearly.

(11.) If the two vessels described in Example 7 were of the same size and were put in communication with each

other, what would be the pressure of the mixed air at the temperature of melting ice?

Answer. 72·626 centimetres.

(12.) The air in a spherical globe of 30 centimetres diameter is compressed into another globe of 15 centimetres diameter, and the temperature is raised from 10° C. to 25° C. Compare the pressures of the air in the two cases, and also compare the pressures on the surfaces of the two globes.

By the combination of the laws of Boyle and Charles we have

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1},$$

$$\therefore \frac{P_2}{P_1} = \frac{V_1}{V_2} \times \frac{T_2}{T_1} = \left(\frac{30}{15}\right)^3 \times \frac{273 + 25}{273 + 10}$$

$$= 8 \times \frac{298}{283} = 8{\cdot}424.$$

Also $\dfrac{\text{whole pressure on surface of 2nd globe}}{\text{whole pressure on surface of 1st globe}}$

$$= \frac{P_2 \times \text{area of surface of 2nd globe}}{P_1 \times \text{area of surface of 1st globe}}$$

$$= \frac{8{\cdot}424}{1} \times \frac{1}{4}$$

$$= 2{\cdot}106.$$

(13.) The lid of a Papin's digester has a safety valve in the form of a frustrum of a cone, the diameter of the bottom of which is one centimetre. The space below the lid contains saturated steam and water. The pressure of the atmosphere is equal to 76 centimetres of mercury pressure, and it is required to raise the temperature of the water to 200° C., at which temperature the pressure of saturated steam is equal to that of 11·689 metres of mercury. What pressure must be applied to the valve?

Neglecting the weight of the valve itself, which is inconsiderable, the mass which must be placed on the valve to keep it down must be at least equal to

$$\frac{\pi}{4} \times 13{\cdot}596\,(1168{\cdot}9 - 76) = 11670{\cdot}3 \text{ grammes,}$$

$$= 11{\cdot}67 \text{ kilogrammes approximately.}$$

N.B. Papin was a philosopher who lived at Marburg during the middle of the seventeenth century.

(14.) What space will be occupied by 3·6 grammes of oxygen when the thermometer is at 23° C. and the barometric height is 74·2 centimetres?

By the formula in Example 12 we have

$$v_2 = v_1 \times \frac{p_1}{p_2} \times \frac{T_2}{T_1}$$

and from the data in Example 12, p. 68, we find that when $p_1 = 76$ and $T_1 = 273$, that $v_1 = \frac{3{\cdot}6}{{\cdot}00143}$ cubic centimetres.

$$\text{Hence } v_2 = \frac{3{\cdot}6}{{\cdot}00143} \times \frac{76}{74{\cdot}2} \times \frac{296}{273} = 2795{\cdot}8 \text{ c.c.}$$

$$= 2{\cdot}7958 \text{ litres.}$$

(15.) What space will one gramme of hydrogen occupy at 14° C. and 73 centimetres of barometric pressure?

Answer. 12·2152 litres.

(16.) At the standard pressure a gramme of a certain gas at 180° C. measured ·3 of a litre, and a gramme of another gas at 83° C. measured ·5 of a litre. Compare the masses of equal volumes of these two gases at 100° C.

Answer. $\dfrac{\text{Mass of 1 litre of first gas}}{\text{Mass of 1 litre of second gas}} = \dfrac{2{\cdot}12}{1}.$

(17.) What is the ratio of the mass of one litre of dry air at 15° C. and 76 centimetres pressure to that of two litres of dry air at 200° C. and 50 centimetres pressure?

Answer. 5 : 4 nearly.

(18.) At what temperature will the density of oxygen gas at 56 centimetres pressure be the same as that of hydrogen at 0° C. and 200 centimetres of barometric pressure?

Answer. 947° C. nearly.

(19.) A glass globe is filled with 8 litres of oxygen at 0° C. and 76 centimetres pressure, and the temperature is then raised to 16° C. while the pressure falls to 75 centimetres. What quantity of oxygen will escape?

Space occupied by the air after expansion

$$= 8 \times \frac{289}{273} \times \frac{76}{75} = 8{\cdot}5818 \text{ litres.}$$

Capacity of the globe after expansion

$$= 8 \left\{ 1 + \frac{16}{38536} \right\} = 8{\cdot}0033 \text{ litres.}$$

∴ volume of oxygen which escapes = ·5785 litre = 578·5 c.c.

But the density of oxygen gas at 16° C. and 75 centimetres pressure is $\cdot 00143 \times \frac{273}{289} \times \frac{75}{76}$, hence the quantity of oxygen which escapes is $578{\cdot}5 \times \cdot 00143 \times \frac{273}{289} \times \frac{75}{76} =$ ·77117 gramme.

(20.) A bottle is filled with air at 25° C., and at atmospheric pressure which is 76 centimetres of mercury, and it is taken into a room where the temperature is 0° C. If the glass stopper has an area of 3 square centimetres, what force will be needed to withdraw it over and above that which is required to overcome the friction?

Answer. 260·058 grammes weight.

(21.) If one litre of dry air at the standard temperature and pressure contains 1·293 grammes, what quantity will there be in one litre at 80° C. and 142 centimetres pressure?

Answer. 1·868 grammes.

(22.) At what temperature will 4·48 grammes of dry hydrogen gas occupy 100 litres at 76 centimetres pressure?

From Example 13, p. 68, we find that at standard temperature and pressure 4·48 grammes of hydrogen will occupy $\frac{4{\cdot}48}{\cdot 0896}$ litres.

And from the formula in Example 12, p. 73, we have

$$\frac{T_2}{T_1} = \frac{V_2}{V_1};$$

$$\text{where } V_2 = 100 \left.\begin{matrix} \\ V_1 = \dfrac{4\cdot48}{\cdot0896} \end{matrix}\right\} \text{ and } T_1 = 273.$$

$$\therefore T_2 = 273 \times \frac{100 \times \cdot0896}{4\cdot48} = 546°.$$

$\therefore$ temperature required $= 546 - 273 = 273°$ C.

(23.) If the temperature remained constant at 0° C., at what pressure would 4·48 grammes of hydrogen occupy 100 litres?

By the formula in Example 12,

$$P_2 = 76 \times \frac{4\cdot48}{\cdot0896} \times \frac{1}{100} = 38 \text{ centimetres.}$$

(24.) A quantity of gas which was collected in a bell-glass over a mercury pneumatic trough measured 174 c.c. The corrected barometric pressure was 75·2 centimetres, the temperature 18° C., and the level of the mercury in the bell-glass was found by measurement to be 5·4 centimetres above the surface of the mercury in the trough. Find the volume of the gas when reduced to standard temperature and pressure. *Answer.* 149·95 c.c.

BUOYANCY OF THE AIR.

(1.) A military balloon which was used at Souakin in 1885 had a capacity of 198·212 cubic metres, and weighed 40·8 kilogrammes. If it were filled with hydrogen, at 0° C. and 76 centimetres barometric pressure, what would be its buoyancy?

The mass of hydrogen in the balloon

$$= 198\cdot212 \times \cdot0896 \times \frac{273}{305} = 15\cdot894 \text{ kilogrammes.}$$

The mass of air displaced by the balloon

$$= 198{\cdot}212 \times 1{\cdot}293 \times \frac{273}{305} = 229{\cdot}399 \text{ kilogrammes.}$$

$$\therefore \text{ buoyancy} = 229{\cdot}399 - (15{\cdot}894 + 40{\cdot}8).$$

$$= 229{\cdot}399 - 56{\cdot}694 = 172{\cdot}705 \text{ kilogrammes.}$$

$$= \frac{172705}{453{\cdot}6} = 380{\cdot}74 \text{ lbs.} = 27 \text{ stone } 2{\cdot}74 \text{ lbs.}$$

(2.) There are 700 litres of warm air at 80° C. in a boy's fire-balloon, while the temperature of the surrounding air is 15° C., and the barometric pressure 76 centimetres. If the balloon can just rise, what must be its mass?

Answer. 158 grammes nearly.

(3.) What would be the buoyancy of 1,000 litres of hydrogen at 0° C. and 76 centimetres barometric pressure?

Answer. 1,201 grammes.

(4.) The apparent mass of a quantity of platinum when counterpoised by brass weights in a delicate balance was 35 grammes. If the density of platinum be 22, that of brass 8·4, and that of air $\frac{1}{773}$, what would this mass of platinum weigh in vacuo?

Let m = true mass of the platinum in grammes;

then $\frac{m}{22}$ = volume of this mass in cubic centimetres;

$$\therefore \frac{m}{22} \times \frac{1}{773} = \left\{ \begin{array}{l} \text{mass in grammes of the air displaced by} \\ \quad \text{the platinum.} \end{array} \right.$$

Hence the *apparent* mass of the platinum in air

$$= m\left\{1 - \frac{1}{22 \times 773}\right\} \quad . \quad . \quad . \quad . \quad (1).$$

Again, the volume of the brass weights is $\frac{35}{8{\cdot}4}$ c.c.,

and the quantity of air displaced by the brass weights is $\frac{35}{8{\cdot}4} \times \frac{1}{773}$ grammes;

$\therefore$ the *apparent* mass of the brass weights

$$=35\left\{1-\frac{1}{8\cdot4\times773}\right\} \text{ grammes} \quad . \quad . \quad . \quad (2).$$

Since the *apparent* masses of brass and platinum are in equilibrium, equating (1) and (2) we have

$$m\left\{1-\frac{1}{22\times773}\right\}=35\left\{1-\frac{1}{8\cdot4\times773}\right\}$$

whence $m = 35 \times \frac{6492\cdot2}{6493\cdot2} \times \frac{17006}{17005} = 34\cdot9967$ grammes.

(5.) The apparent mass of a quantity of aluminium, whose density is 2·57, is 2 grammes when counterpoised with brass weights. What is its actual mass?

Answer. 2·0006 grammes nearly.

Note.—In practice brass weights are generally employed. With the exception of aluminium, gold, and platinum, the density of the majority of the metals is not very different from that of brass, so that the correction on account of the buoyancy of the air may generally be omitted in weighing *metals* unless very great accuracy is required.

(6.) A certain balance is capable of weighing to $\frac{1}{10}$th of a milligramme, and the weights are of brass, whose density is 8·2. Up to what weight will the effect of the air displaced by the weights be inappreciable?

Let x = required weight in grammes, then $\frac{x}{8\cdot2}$ is the volume of the air displaced by the weights in c.c.s, and at standard temperature and pressure the quantity of air displaced by these weights is $\frac{x\times\cdot001293}{8\cdot2}$ grammes.

But the maximum value of this quantity of air must be not greater than $\frac{1}{10}$th of a milligramme.

$$\therefore \frac{x\times\cdot001293}{8\cdot2} = \cdot0001\,;$$

$$\therefore x = \cdot6342 \text{ gramme nearly.}$$

(7.) If the balance was capable of weighing to centigrammes only, find the maximum weight for which the air displaced by the brass weights would be inappreciable.

Answer. 63·42 grammes.

(8.) It is required to weigh out accurately 230·5 grammes of mercury. The density of mercury being 13·596, what must be the indicated value of the brass weights to be placed in the other scale-pan?

Let x be the required value of the brass weights in grammes; then, since mercury is more dense than brass, the mass of the air displaced by the brass will exceed that of the air displaced by the mercury by a quantity which is very approximately equal to

$$x \times \cdot 001293 \left\{ \frac{1}{8\cdot 4} - \frac{1}{13\cdot 596} \right\} = x \times \cdot 0000588 \text{ grammes,}$$

and therefore

$$230\cdot 5 = x\{1 - \cdot 0000588\}$$

$$\text{whence } x = 230\cdot 5136 \text{ grammes.}$$

(9.) The density of ice being ·92, what brass weights must be used in order to weigh out 200 grammes of ice, allowing for the buoyancy of the air and taking the density of brass at 8·4? *Answer.* 199·75 grammes.

N.B. The student should notice that when the density of the substance is less than that of the material of the weights, the true mass is greater than the apparent mass, and *vice versâ.*

(10.) A quantity of mercury weighed 75·261 grammes in air when counterpoised with platinum weights. What was its true mass? *Answer.* 75·264 grammes, very nearly.

(11.) A piece of oak and a piece of platinum counterpoised each other in the scale-pans of an accurate balance. Compare their masses, taking into account the buoyancy of the air, and assuming the density of platinum to be 22, that of oak ·69, and that of air ·0013.

Answer. $\dfrac{\text{Mass of the oak}}{\text{Mass of platinum}} = 1\cdot 00183.$

(12.) At the standard temperature and pressure two glass flasks, which displaced 13 and 6 litres of air, equilibrated each other in a sensitive balance. After awhile the temperature of the air changed to 25° C., and the pressure became 75 centimetres. What weight had to be added to one of the scale-pans so as to restore the equilibrium?

Let m = mass of the larger flask in grammes.
m' = „ smaller „ „

At the standard temperature and pressure the quantity of air displaced by the larger flask is $13 \times 1{\cdot}293 = 16{\cdot}809$ grammes, and by the smaller flask is $6 \times 1{\cdot}293 = 7{\cdot}758$ grammes.

Hence the apparent mass of the larger flask $= m - 16{\cdot}809$ grammes, and the apparent mass of the smaller flask $= m' - 7{\cdot}758$ grammes; and as they equilibrate each other at this temperature and pressure,

$$m - 16{\cdot}809 = m' - 7{\cdot}758;$$
$$\therefore\ m - m' = 9{\cdot}051 \text{ grammes} \quad . \quad . \quad . \quad . \quad (1).$$

When the temperature has risen to 25° C., and the barometric pressure has changed to 75 centimetres, the volume of the larger flask is

$$13\left(1 + \frac{25}{38536}\right) = 13{\cdot}008 \text{ litres,}$$

and the volume of the smaller flask is

$$6\left(1 + \frac{25}{38536}\right) = 6{\cdot}004 \text{ litres;}$$

also the mass of one litre of air is

$$1{\cdot}293 \times \frac{273}{298} \times \frac{75}{76} = 1{\cdot}169 \text{ grammes nearly,}$$

and $\therefore$ the quantity of air displaced by large flask

$$= 13{\cdot}008 \times 1{\cdot}169 = 15{\cdot}206 \text{ grammes,}$$

and the quantity of air displaced by small flask

$$= 6{\cdot}004 \times 1{\cdot}169 = 7{\cdot}019 \text{ grammes.}$$

Hence the apparent mass of the large flask $= m - 15{\cdot}206$ grammes, and the apparent mass of the small flask$= m' - 7{\cdot}019$ grammes.

The apparent mass of the large flask exceeds that of the small flask by $m - m' - (15{\cdot}206 - 7{\cdot}019) = 9{\cdot}051 - 8{\cdot}187 = {\cdot}864$ gramme, and $\therefore$ ·864 gramme must be added to the small flask so as to restore the equilibrium.

(13.) A closed vessel displaces 10·5 litres of air, and is equilibrated in a balance by weights whose volume is inconsiderable as compared with that of the vessel. The balance is in equilibrium when the barometer stands at 72 centimetres. If the barometer rise to 76·2 centimetres, the temperature in both cases being 15° C., what mass must be added to restore the equilibrium?

Answer. ·7507 gramme.

(14.) Find the absolute density of dry air at 0° C. and 76 centimetres pressure from the following data, which were obtained by Regnault by his *compensation* method.

When the globe was filled with dry air at 0° C. and 76·199 centimetres pressure and 1·487 gramme added, they equilibrated the compensation globe and the counterpoise.

After being placed in melting ice, and exhausted till the pressure of the residual air was ·843 centimetre, the globe and its contents, together with 14·151 grammes, equilibrated the compensation globe and counterpoise.

At 6° C. and 76·177 centimetres pressure the globe, when open, weighed 1258·55 grammes.

When it was filled with water at 0° C., and weighed at 6° C. and 76·177 centimetres pressure, it weighed 11126·06 grammes.

For a full description of this method see Deschanel's 'Natural Philosophy,' par. 221.

Let M = mass of the globe in grammes.

M' = „ water filling globe at 0° C.

m = „ compensating globe.

m' = „ counterpoise.

Let V = capacity of globe at 0° C. in cubic centimetres.
d = absolute density of air at 0° C. and 76 centimetres pressure—*i.e.* the number of grammes in a cubic centimetre.

To find Vd, or the quantity of air which fills the globe at 0° C. and 76 centimetres of pressure, we have the two equations

$$\left.\begin{array}{l} M + Vd \times \frac{76 \cdot 119}{76} + 1 \cdot 487 = m + m' \\ M + Vd \times \frac{\cdot 843}{76} + 14 \cdot 151 = m + m' \end{array}\right\}$$

whence we have Vd = 12·786 grammes nearly . . . . (1).

We have next to find V, *i.e.*, the capacity of the globe at 0° C. and 76 centimetres pressure. From the second set of weighings we have

The apparent mass of the water in globe

$$= 11126 \cdot 06 - 1258 \cdot 55 = 9867 \cdot 51 \text{ grammes.}$$

But the mass of the air displaced by the water filling the globe when it was weighed at 6° C. and 76·177 centimetres pressure was

$$12 \cdot 786 \times \frac{273}{279} \times \frac{76 \cdot 177}{76} = 12 \cdot 54 \text{ grammes,}$$

and the true mass of the water is equal to the apparent mass *plus* the mass of the air displaced,

$$\therefore M' = 9867 \cdot 51 + 12 \cdot 54 = 9880 \cdot 05 \text{ grammes.}$$

Taking the density of water at 4° C. as unity, that of water at 0° C. is ·999881, and therefore the capacity of the globe at 0° C. is

$$V = \frac{9880 \cdot 05}{\cdot 999881} = 9881 \cdot 2 \text{ cubic centimetres.}$$

But $\qquad Vd = 12 \cdot 786$ grammes.

∴ Absolute density of air at 0° C. and 76 centimetres pressure is

$$d = \frac{12 \cdot 786}{9881 \cdot 2} = \cdot 001294 \text{ gramme.}$$

(15.) Find the density of carbonic acid gas from the following data obtained by Regnault by his compensation method.

When the globe was filled with dry air at 0° C. and 74·721 centimetres pressure, and 1·699 gramme added, they equilibrated the compensation globe and counterpoise.

After being placed in melting ice and exhausted till the pressure of the residual air was ·756 centimetre, a mass of 14·1345 grammes had to be added so as to equilibrate the compensation globe and counterpoise. When the globe was filled with carbonic acid gas at 0° C. and 75·634 centimetres pressure, the additional mass required was ·808 gramme. After exhaustion at 0° C. till the pressure of the residual gas was ·171 centimetre, the additional mass required was 20·2085 grammes.

Answer. Density of carbonic acid (air = 1) = 1·5291.

(16.) In an experiment to determine by Regnault's compensation method the density of dry air a globe was used whose capacity was exactly 10 litres. When it was filled with dry air at 18° C. and 75·4 centimetres pressure, it weighed 12·01 grammes more than it did after so much air had been pumped out that the pressure-gauge only indicated ·5 centimetre of mercury pressure. Taking the coefficient of expansion of the glass at $\frac{1}{38700}$, find from the data of this experiment the absolute density of dry air at 0° C. and 76 centimetres pressure.

Answer. ·001298 gramme nearly.

(17.) Find the apparent mass of 4 kilogrammes of gold when weighed (1) in water, and (2) in air, the densities at the temperature of the experiment being respectively

For gold . . .	19·4
„ water . .	1·0
„ air . . .	·001293
„ the weights	8·4

Answers. (1) 3794·411 grammes ; (2) 4000·356 grammes

(18.) At a certain place the limits of range of the barometric pressure are from 71·3 to 78·1 centimetres of mercury, and the limits of temperature $-19°$ C. and $+36°$ C. What will be the maximum variation in the apparent mass of a kilogramme of copper, the density of copper at 0° C. being 8·86 and its coefficient of cubical expansion $\frac{1}{19400}$?

Answer. ·0408 of a gramme.

THE BOILING POINT AND HYPSOMETRY.

(1.) The position of the top of the mercury column in a delicate thermometer when exposed to the vapour of boiling water was marked 100° C., the barometric pressure at the time, when reduced to 0° C., being 74·65 centimetres. The stem was 30 centimetres long and covered a range of 30° C. What would be the true position of the boiling point?

The temperature at which a liquid boils is that temperature at which the maximum pressure of the vapour of the liquid is equal to the external pressure upon the liquid. For the position of the mark 100° C. for the boiling point of water to be correct, the pressure of the atmosphere at the time of the experiment must be equivalent to a barometric column of 76 centimetres at 0° C.

Let x be the true distance (in centimetres) of the top of the mercury column from the position in the tube where the zero point would be if the thermometer were sufficiently prolonged. Then on referring to a table of vapour pressures we find that the temperature 99·5° C. corresponds to a maximum vapour pressure of 74·65 centimetres.

$$\therefore \frac{x}{100} = \frac{100}{99·5} \quad \therefore x = 100·503$$

Hence the corrected position of the boiling point is 5 millimetres *above* the indicated one.

(2.) The position of the boiling point of a delicate thermometer was determined when the height of the barometric column was 76·82 centimetres, and each centimetre of the tube corresponded to 1° Centigrade of temperature difference. What would be the corrected position of the 100° C. mark?

Answer. 3 millimetres *below* the indicated mark.

(3.) When the position of the boiling point of a certain thermometer was determined and marked, the barometer stood at 73·85 centimetres. The distance between the indicated boiling point and the freezing point was 30 centimetres. How far would the true position of the 100° C. mark be above the indicated one?

Answer. 2·4 millimetres.

(4.) At the sea-level and when the barometric pressure is equal to 76 centimetres of mercury, water boils at the temperature of 100° C. At what temperature would water boil at the top of Mont Blanc, where the barometric pressure is 41·7 centimetres?

From a table of vapour pressures we find that

At 80° C. the maximum pressure of aqueous vapour is 35·4643 centimetres.

At 85° C. the maximum pressure of aqueous vapour is 43·341 centimetres,

and ∴ the temperature corresponding to a maximum pressure of aqueous vapour equal to 41·7 centimetres is

$$80 + \frac{5 \times 62 \cdot 357}{78 \cdot 767} = 80 + 3 \cdot 958 = 84^\circ \text{ C. nearly.}$$

(5.) It is an established fact that for small variations of temperature on either side of 100° C. a difference of pressure of 2·7 centimetres of mercury corresponds to a variation of 1° C. in the temperature at which ebullition commences. If the annual range of barometric pressure be from 72 to 79 centimetres, what will be the corresponding variation in the temperature at which water boils?

Let x° C. be the temperature at which ebullition begins.

d = difference of actual pressure from that of 76 centimetres.

Then we have $x = 100 \pm \frac{d}{2{\cdot}7}$.

In the present case $d = +3$ and $d = -4$, and

$$\therefore x_1 = 100 + \frac{3}{2{\cdot}7} = 101\tfrac{1}{9}^\circ \text{ C.}$$

$$x_2 = 100 - \frac{4}{2{\cdot}7} = 98\tfrac{13}{27}^\circ \text{ C.}$$

(6.) Having given that the maximum pressure of aqueous vapour at 34° C. is 39·565 millimetres, and at 35° C. is 41·827 millimetres, what will be the temperature at which water will boil under the receiver of an air-pump when the pressure is 40 millimetres? *Answer.* 34·19° C.

(7.) Some water is placed in a saucer under the receiver of an air-pump, and the temperature of the air is 0° C. At what pressure will the water boil?

The maximum pressure of aqueous vapour at 0° C. is 4·6 millimetres of mercury, and this is therefore the pressure at which water boils when at the freezing temperature.

(8.) The steam in a boiler is at the pressure of 30 lbs. to the square inch. What is the temperature of the water?

A pressure of 14·7 lbs. to the inch corresponds to 76 centimetres of mercury pressure.

$\therefore$ a pressure of 30 lbs. to the inch corresponds to $\frac{30}{14{\cdot}7} \times 76$ $= 155{\cdot}1$ centimetres.

But for 120° C. the maximum pressure of aqueous vapour is 149·128 centimetres, and at 125° C. it is 169·076 centimetres, $\therefore$ the temperature required is

$$120 + \frac{5 \times 5{\cdot}972}{19{\cdot}948} = 120 + 1{\cdot}497$$
$$= 121{\cdot}5^\circ \text{ C. nearly.}$$

(9.) The temperature of the steam in a high-pressure boiler is 150° C.; what is the pressure of the steam in pounds per square inch? *Answer.* 69·27 lbs. approximately.

(10.) The temperature of the steam in a locomotive boiler is 170° C., and the pressure of saturated steam at that temperature is 596·166 centimetres of mercury. What is the pressure in pounds per square inch?

Answer. 115·3 lbs. nearly.

(11.) The temperature at which water boils on the top of the Finsteraarhorn is found to be 86·1° C. Deduce from this the height of the mountain in metres.

If we only require an approximately correct result we may use Soret's formula

$$\begin{aligned} h &= 295\ (100 - t) \text{ metres} \\ &= 295\ (100 - 86{\cdot}1) = 295 \times 13{\cdot}9 \\ &= 4100{\cdot}5 \text{ metres} \\ &= 4100{\cdot}5 \times 3{\cdot}2809 = 13453 \text{ feet.} \end{aligned}$$

N.B. The true height is 14,100 feet.

(12.) Find the height of the following places above the sea-level :—

Place	Boiling point of water
Moscow	99° C.
Quito	90·1°
Hospice of the St. Gothard	92·9°
Antisana (South America)	86·3°

Answers.	Moscow	967·9 feet
	Quito	9582·2 „
	St. Gothard	6872·0 „
	Antisana	13260·0 „

(13.) By means of the hypsometer the boiling point at the lower of two stations was found to be 99·5° C., and at the upper one 97° C. What was the difference of level of the two stations? *Answer.* 2,420 feet approximately.

N.B. The student must notice that the hypsometer only gives the temperature of the boiling point, and that the atmospheric pressure is deduced from this by referring to a table of vapour pressures. The subsequent computations for obtaining a formula which shall give the height are the same as when a barometer is used.

(14.) The height of the barometric column, reduced to 0° C., was 76 centimetres at the foot of a mountain where the temperature of the air was 18° C., while at the top of the mountain the corrected height of the barometer was 70 centimetres and the air temperature 12° C. Find the height.

In order to obtain a general formula for the determination of the height of a mountain by means of barometrical observations, several assumptions have to be made, and the resulting formula is very complicated. For heights up to 1,200 metres the following formula, which is known as Babinet's, answers fairly well.

$$x = 16000 \frac{\text{H} - h}{\text{H} + h} \left\{ 1 + \frac{2(t + t')}{1000} \right\} \text{metres}$$

where H and h are the corrected heights of the barometer at the two stations, and t t' are the temperatures of the air in Centigrade degrees.

Substituting the numbers given in the question we find that

$$x = 16000 \times \frac{6}{146} \left\{ 1 + \frac{2 \times 30}{1000} \right\}$$
$$= 697 \text{ metres approximately.}$$

(15.) The following observations were made to determine the difference of level of two stations.

	Lower station	Upper station
Barometric height in centimetres .	73·65	71·69
Temperature of the air	9·75° C.	7·75° C.

Find the difference of level by Babinet's formula.

Answer. 223·3 metres nearly.

(16.) Find the height of Arthur's Seat from the following observations. At Leith pier the height of the barometer was 75·1 centimetres, and the thermometer 12·2° C. On the summit of Arthur's Seat the barometer indicated 72·9 centimetres while the thermometer stood at 10·1° C.

Answer. 248·45 metres approximately.

VAPOUR PRESSURE AND HYGROMETRY.

(1.) A long barometer tube, inverted in a deep mercury trough and supported in a vertical position, contained some dry air which occupied a length of 12 centimetres when the surface of the mercury in the tube was 50 centimetres above that in the trough, the atmospheric pressure being 75 centimetres. A little ether was passed up into the air space, and the tube was depressed until the length occupied by the mixture of air and ether vapour was 12 centimetres. It was then found that the length of the mercury column was 21·36 centimetres. What was the pressure of the ether vapour?

Since the air occupies the same space in both cases its pressure is the same, and is equal to $75 - 50 = 25$ centimetres of mercury pressure. Also the pressure of the mixture is equal to the sum of the pressures of the ether vapour and of the air. Hence if x be the pressure of the ether vapour

$x + 25 = 75 - 21{\cdot}36 \therefore x = 28{\cdot}64$ centimetres of mercury.

(2.) A similar experiment was made with dry air, which at first occupied a length of 15 centimetres of the inverted tube while the mercury column was 52 centimetres long, the barometric pressure being 76 centimetres. On introducing a little carbon bisulphide and depressing the tube in the trough until the mixture occupied 15 centimetres, the height of the mercury column in the tube was 31·7 centi-

metres. The temperature was the same as in the last example, viz. 10° C. What was the pressure of the vapour?

Answer. 20·3 centimetres of mercury.

(3.) Two inverted barometer tubes were placed vertically side by side, and the height of the mercury column in each was observed by the aid of a cathetometer, and found to be 74·3 centimetres. A few drops of water were passed up into one tube and a few drops of ether into the other, and the height of the mercurial column in the ether tube was then found to be 38·94 centimetres, and in the water tube 73·03 centimetres, the temperature of the room being 15° C. Find the pressures of these vapours at this temperature.

Answers. Pressure of water vapour = 1·27 centimetres.
" ether " = 35·36 "

(4.) These two tubes were then enclosed according to Dalton's method in a wide glass cistern containing water which was heated to 30° C., and it was found that the level of the top of the mercury column in the tube containing water vapour had fallen to 71·84 centimetres, while that of the mercury in the other tube had fallen to 11·52 centimetres. The laboratory barometer standing at 75 centimetres, find the pressure of saturated ether vapour, and also that of water vapour at 30° C.

Answers. For ether vapour 63·48 centimetres.
" water " 3·16 "

(5.) A large glass vessel contains a quantity of dry air at 100° C. and 76 centimetres pressure, and water is then allowed to enter the vessel until it ceases to evaporate. What is the final pressure of the mixture?

When a liquid is introduced into a confined space which already contains a gas upon which the liquid does not act chemically, as much vapour is formed as if the gas were not present, and the vapour attains to its maximum pressure corresponding to the particular temperature. The gas is then saturated with the vapour, and the actual pressure of the mixture is equal to the pressure which the gas would

exert if it alone occupied the whole space *plus* the maximum pressure of the vapour corresponding to the temperature of the mixture.

In the present case, the maximum pressure of aqueous vapour at 100° C. being 76 centimetres of mercury, the pressure of the mixture, expressed in terms of the equivalent mercury column, is

$$76 + 76 = 152 \text{ centimetres.}$$

(6.) A long vertical barometer tube is inverted over mercury and contains a quantity of dry air which, at the temperature of 20° C., occupies a length of 20 centimetres in the tube, and the level of the mercury in the tube is 56 centimetres above that in the cistern, the laboratory barometer standing at 76 centimetres. A small quantity of water is then passed up into the tube and the mercury column falls to 55·13 centimetres. Find from these data the pressure of aqueous vapour at 20° C.

The original pressure of the dry air is $76 - 56 = 20$ centimetres. When the air has become saturated with moisture it occupies a length of 20·87 centimetres, and therefore its final pressure

$$= 20 \times \frac{20}{20{\cdot}87} = 19{\cdot}12 \text{ centimetres.}$$

But the atmospheric pressure balances the pressure of this air, the pressure of the vapour, and the column of mercury in the tube; hence, if x be the vapour pressure we have

$$x + 55{\cdot}13 + 19{\cdot}12 = 76$$
$$\therefore\ x = 1{\cdot}75 \text{ centimetre.}$$

(7.) A similar experiment was then performed with ether vapour. The air initially occupied 18 centimetres of the tube and the height of the mercury column was 58 centimetres, the laboratory barometer standing at 75 centimetres. On introducing a little ether the height of the mercury column became 25·7 centimetres. What was the pressure of the ether vapour? *Answer.* 43·217 centimetres.

(8.) A long barometer tube of uniform bore which is inverted over mercury contains at its upper part some dry air which occupies a length of 15 centimetres. The height of the mercury column in the tube is 50 centimetres, the barometric pressure being 74·5 centimetres. A little alcohol is passed up into the tube, and it is known that at the temperature of the experiment the maximum pressure of alcohol vapour is 7·85 centimetres. What should be the height of the mercury column in the tube?

Let x be the depression of the mercury column in centimetres. Then by the method of Example 6 we find that the final pressure of the air is $24{\cdot}5 \times \frac{15}{15+x}$ centimetres, and the height of the mercury column is $50-x$ centimetres. Hence we have

$$7{\cdot}85 + \frac{24{\cdot}5 \times 15}{15+x} + 50 - x = 74{\cdot}5\,;$$

and solving this quadratic equation we obtain

$$x = 3{\cdot}36 \text{ centimetres.}$$

∴ height of mercury column $= 50 - x = 46{\cdot}64$ centimetres.

(9.) Three cubic metres of moist air were drawn through a chemical hygrometer, consisting of a series of U tubes containing pumice-stone soaked in strong sulphuric acid, and 34·68 grammes of water were deposited in them, the temperature of the room being 18° C. Knowing that at 18° C. the maximum pressure of aqueous vapour is 1·535 centimetres of mercury, find from these data the *hygrometric state* of the air in the room.

The quantity of aqueous vapour actually contained in one cubic centimetre of air is called its *absolute humidity*. The ratio which this bears to the quantity of aqueous vapour which would be contained in a cubic centimetre if the air were saturated is called the *hygrometric state.*

Proceeding as in Example 11, we shall find that the quantity of aqueous vapour which would be contained in the

given volume of air if it were saturated at 18° C. is 45·94 grammes approximately,

$$\therefore \text{ the } \textit{hygrometric state} = \frac{34{\cdot}68}{45{\cdot}94} = {\cdot}75 \text{ approximately.}$$

(10.) On another occasion one cubic metre of moist air was drawn through the apparatus and 4·29 grammes of water were collected by the pumice. The maximum pressure of aqueous vapour at the temperature of the room, which was 20° C., is 1·739 centimetres. What was the *hygrometric state* of the air. *Answer.* ·25 approximately.

(11.) Find the quantity of aqueous vapour which is contained in a cubic metre of air which is fully saturated at 20° C.

A space full of air or any gas which is saturated with vapour contains as much vapour as if there were no air or other gas present.

At the temperature of 20° C. and 1·739 centimetres of pressure, the quantity of dry air in a cubic metre is

$$1293 \times \frac{273}{293} \times \frac{1{\cdot}739}{76} \text{ grammes.}$$

And as the density of aqueous vapour is always $\frac{5}{8}$ths of the density of dry air at the same temperature and pressure, the required quantity of aqueous vapour is

$$\frac{5}{8} \times 1293 \times \frac{273}{293} \times \frac{1{\cdot}739}{76} = 17{\cdot}229 \text{ grammes.}$$

(12.) What quantity of aqueous vapour is contained in the air of a room which is 9 metres long, 6 metres wide, and 4 metres high, the temperature being 15° C. and the air only half-saturated? The maximum pressure of aqueous vapour at 15° C. is 1·27 centimetres.

Answer. 1·3824 kilogramme nearly.

N.B. In this example the student must notice that the density of the aqueous vapour is $\frac{5}{16}$ (not $\frac{5}{8}$) that of dry air, because the hygrometric state is $\frac{1}{2}$.

(13.) Find the quantity of aqueous vapour which is con-

tained in a cubic metre of air, the dew-point being at 12° C., and the maximum pressure of aqueous vapour at 12° C. being 1·046 centimetre.

The 'dew-point' is that temperature at which the air would be fully saturated by the quantity of vapour which it actually contains. Proceeding as in the solution of Example 11, we find that the quantity of vapour is

$$\frac{5}{8} \times 1293 \times \frac{273}{285} \times \frac{1·046}{76} = 10·654 \text{ grammes.}$$

(14.) The temperature of the air in a laboratory was observed to be 22° C. and that of the dew-point 13° C. Knowing that the maximum pressures of aqueous vapour corresponding to these two temperatures are 1·965 and 1·116 centimetre respectively, find the hygrometric state of the air. *Answer.* ·586 nearly.

(15.) The dew-point is at 12° C. and the temperature of the air in a room is 17° C.; find the hygrometric state of the air, the maximum pressure of aqueous vapour at these temperatures being 1·046 and 1·442 centimetre respectively. *Answer.* ·74 nearly.

N.B. The student should notice that the dampness of the air does not depend upon the quantity of water vapour actually contained in a given volume of the air, but upon the ratio which this quantity bears to the quantity which would be required in order to saturate the air at the particular temperature.

(16.) A closed inextensible glass vessel contains a quantity of air at the temperature of 12° C., the hygrometric state of which is ·72. What will be the hygrometric state of the air if its temperature be raised to 30° C., the maximum pressure of aqueous vapour at 30° C. being 3·16 centimetres? *Answer.* ·24 nearly.

(17.) The air in a room is at the temperature of 25° C. and its hygrometric state is ·8. Assuming that the maximum pressure of aqueous vapour at 25° C. is 2·36 and at

5° C. is ·65 centimetre of mercury, find the quantity of vapour per cubic metre that will be deposited if the temperature fall to 5° C.

Let m = number of grammes of water vapour per cubic metre of air at 25° C.

m' = number of grammes of water vapour per cubic metre of air saturated at 5° C.;

then $m = \cdot 8 \times \frac{5}{8} \times 1293 \times \frac{2\cdot 36}{76} \times \frac{273}{298} = 18\cdot 39$ grammes.

$$m' = \frac{5}{8} \times 1293 \times \frac{\cdot 65}{76} \times \frac{273}{278} = 6\cdot 79 \text{ grammes nearly.}$$

And the quantity of aqueous vapour per cubic metre which is deposited in the form of dew is

$$m - m' = 18\cdot 39 - 6\cdot 79 = 11\cdot 6 \text{ grammes.}$$

(18.) Assuming that a portion of the atmosphere, 1,000 feet high, and extending over a space of 1 mile square, is at a uniform temperature of 20° C., and is fully saturated with vapour, how much rain will be deposited from this when the temperature falls to 10° C., the maximum pressure of aqueous vapour at 20° C. being 1·74 centimetre and at 10° C. being ·92 centimetre? *Answer.* 6061·64 tons.

N.B. One cubic foot = 28·316 litres.
One ton = $1\cdot 01605 \times 10^6$ grammes.

(19.) If instead of the air being saturated with vapour its dew-point had been at 15° C. while the temperature was 20° C., what would have been the quantity of rain, the maximum pressure of aqueous vapour at 15° C. being 1·27 centimetre? *Answer.* 2613·6 tons.

(20.) When the temperature of the room is at 16° C. and the barometric pressure 76·2 centimetres, 2·5 grammes of air saturated with moisture are measured off, and it is required to know how much dry air there is in this, the maximum pressure of aqueous vapour at 16° C. being 1·353 centimetre of mercury.

Let v = number of litres in the mixture of air and water vapour.

x = number of grammes of water vapour in the mixture.

$$\text{Then } x = \frac{5}{8} \times v \times 1{\cdot}293 \times \frac{1{\cdot}353}{76} \times \frac{273}{289} \text{ grammes} \quad . \quad . \quad (1)$$

And as the pressure of the air is $76{\cdot}2 - 1{\cdot}353 = 74{\cdot}847$ centimetres, the mass of dry air is

$$2{\cdot}5 - x = v \times 1{\cdot}293 \times \frac{74{\cdot}847}{76} \times \frac{273}{289} \text{ grammes} \quad . \quad . \quad (2)$$

From (1) and (2) we obtain by division

$$\frac{2{\cdot}5 - x}{x} = \frac{74{\cdot}847}{1{\cdot}353} \times \frac{8}{5} = 88{\cdot}511,$$

whence $x = {\cdot}0278$ gramme;

$\therefore$ mass of dry air $= 2{\cdot}5 - {\cdot}0278 = 2{\cdot}4722$ grammes.

(21.) A quantity of gas saturated with moisture was collected over mercury in a vertical glass tube. The gas occupied 260 c.c.s, and the upper level of the mercury was 53·3 centimetres above the mercury in the trough. The laboratory barometer indicated 75·4 centimetres of pressure, and the temperature was 13° C. The maximum pressure of aqueous vapour at 13° C. being 1·116 centimetre, find the volume of dry air reduced to what it would be at 0° C. and 76 centimetres of pressure. *Answer.* 68·52 c.c.s.

(22.) A quantity of moist gas was collected in a bell-glass over a water pneumatic trough. Find from the following data the volume of dry gas at standard pressure and temperature.

Observed volume of moist gas .	=	230 c.c.
Barometric pressure, corrected and reduced	=	74·2 centimetres.
Height of water column . .	=	16·0 „
Maximum pressure of aqueous vapour at 15° C. . . .	=	1·27 „
Temperature of room . . .	=	15° C.
Density of water at 15° C. . .	=	·99915.

The height of the mercury column, the pressure of which at 0° C. is equal to that of the water column, is

$$\frac{16 \times \cdot 99915}{13 \cdot 596} = 1 \cdot 176 \text{ centimetre,}$$

and therefore the pressure due to the gas alone is

$$74 \cdot 2 - (1 \cdot 176 + 1 \cdot 27) = 71 \cdot 754 \text{ centimetres.}$$

Hence the volume of the dry gas when reduced to standard pressure and temperature is

$$230 \times \frac{71 \cdot 754}{76} \times \frac{273}{288} = 205 \cdot 84 \text{ c.c.}$$

(23.) From the following data calculate the volume of dry gas at the standard pressure and temperature.

Observed volume of moist gas	= 137·2 c.c.
Height of mercury in tube above that in the trough	= 14·75 centimetres.
Height of the barometer	= 73·85 "
Maximum pressure of aqueous vapour at 15° C.	= 1·27 "
Temperature of room	= 15° C.

Answer. 98·96 c.c.

(24.) When the barometer stood at 75·4 centimetres, and the thermometer at 13° C., a quantity of gas saturated with moisture was collected over mercury and found to occupy 360 c.c. The level of the mercury in the tube being 44 centimetres above that in the trough, and the maximum pressure of aqueous vapour at 13° C. being 1·116 centimetres, what was the volume of dry gas when reduced to standard pressure and temperature?

Answer. 136·93 c.c.

Note.—When gases are collected in this way their hygrometric state hardly ever corresponds to that of complete saturation, and as it is practically impossible to determine its actual value, it is best to insure saturation by passing a few drops of water up the tube by means of a curved pipette.

Or we may allow for the humidity by remembering that in practice the densities of liquids are generally determined at temperatures which range between 15° C. and 20° C., and that the pressure due to the aqueous vapour never amounts to more than a few millimetres, so that we may take it at half that corresponding to 15° C. or 20° C., that is to say, at ·63 or ·87 of a centimetre.

(25.) What percentage of error would occur in the result of the last example if the correction for the humidity of the gas were neglected?

The pressure of the gas would be $74·5 - 44 = 30·5$ centimetres, and the reduced volume of the air would be

$$360 \times \frac{30·5}{76} \times \frac{273}{286} = 137·91 \text{ c.c. nearly;}$$

and therefore the error $= 137·91 - 136·93 = ·98$ c.c.

$$\therefore \text{ percentage error} = \frac{·98}{136·93} = ·007157$$
$$= ·72 \text{ per cent. nearly.}$$

(26.) A room contains 64 cubic metres of dry air at 8° C., and a barometric pressure of 76 centimetres. If 500 grammes of water are allowed to evaporate into the air, what will be the pressure of the vapour?

Let x be the pressure expressed in centimetres of mercury; then, since the 500 grammes of water-vapour occupy 64 cubic metres at 8° C., we have

$$500 = \frac{5}{8} \times 64 \times 1293 \times \frac{273}{281} \times \frac{x}{76},$$

whence $x = ·756$ centimetre.

(27.) What would be the hygrometric state of the air in this room, the maximum pressure of aqueous vapour at 8° C. being ·802 centimetre?

The hygrometric state being equal to the ratio of the actual pressure of the aqueous vapour in the air to the pressure which would exist if the air were saturated, is

$$\frac{·756}{·802} = ·942.$$

(28.) A cubic metre of air at 5° C. and 76 centimetres of pressure is in contact with water, and is heated up to 25° C. at constant pressure, so that it is saturated with moisture at both temperatures. The maximum pressure of aqueous vapour at 5° C. and 25° C. being ·65 centimetre and 2·36 centimetres respectively, find the space occupied by the moisture.

When reduced to standard pressure and temperature the cubic metre of moist air at 5° C. would occupy a space

$$v = \frac{76 - \cdot 65}{76} \times \frac{273}{278} = \frac{75 \cdot 35}{76} \times \frac{273}{278} \text{ cubic metre.}$$

But the pressure of the air at 25° C. would be 76 – 2·36 = 73·64 centimetres, and therefore the space occupied by the volume v of dry air when saturated with moisture at 25° C. would be

$$v \times \frac{76}{73 \cdot 64} \times \frac{298}{273} = \frac{75 \cdot 35}{73 \cdot 64} \times \frac{298}{278} = 1 \cdot 0968 \text{ cubic metre.}$$

(29.) A cubic metre of dry air at 50° C. and 76 centimetres of pressure is saturated with moisture. The pressure of aqueous vapour at 50° C. being 9·2 centimetres, find the space occupied by the moist air at the above temperature and pressure. *Answer.* 1·1377 of a cubic metre.

(30.) Four cubic metres of dry air at 15° C. and 76·2 centimetres pressure absorb moisture to such an extent that the hygrometric state becomes ·63. What space will the moist air occupy at the same pressure and temperature, the maximum pressure of aqueous vapour at 15° C. being 1·27 centimetre?

The pressure of the aqueous vapour = ·63 × 1·27 = ·8 centimetre nearly.

∴ pressure of air = 76·2 – ·8 = 75·4 centimetres.

$$\therefore \text{ new volume} = 4 \times \frac{76 \cdot 2}{75 \cdot 4} = 4 \cdot 042 \text{ cubic metres.}$$

(31.) What quantity of aqueous vapour must be added to a metre of dry air at 25° C. and 74 centimetres pressure

so as to produce a hygrometric state of ·75, the maximum pressure of aqueous vapour at 25° C. being 2·36 centimetres?

By the method of Example 30, we find that the pressure of the aqueous vapour is 1·77 centimetre, and that the space occupied by the moist air is 1024·5 litres. Hence the quantity of water absorbed by the air is

$$\frac{5}{8} \times 1024{\cdot}5 \times 1{\cdot}293 \times \frac{1{\cdot}77}{76} \times \frac{273}{298} = 17{\cdot}664 \text{ grammes.}$$

(32.) Supposing that on a fine evening and with a clear sky the radiation from the grass cools it down to 7° C. below that of the surrounding air, which is at the temperature of 14° C., what must be the hygrometric state of the air for there to be a deposition of dew? The maximum pressure of aqueous vapour at 14° C. is 1·19 centimetre of mercury, and at 7° C. it is ·749 centimetre.

Answer. The hygrometric state $= \dfrac{{\cdot}749}{1{\cdot}19} = {\cdot}63$ nearly.

(33.) A barometer tube which is inverted over mercury is surrounded by a hot-air jacket which keeps the mercury in the tube at the temperature of 200° C. The height of the mercury column in the tube is 76·68 centimetres, and the barometric pressure outside is 76 centimetres. What is the pressure of mercury vapour at 200° C.?

The column of mercury which, at 0° C., is equivalent to the column of mercury in the tube, is $\dfrac{76{\cdot}68 \times 5550}{5750} = 74{\cdot}01$ centimetres nearly.

∴ pressure of mercury vapour $= 76 - 74{\cdot}01 = 1{\cdot}99$ centimetre.

(34.) In another experiment the external pressure was 75·21 centimetres, and the jacket surrounding the tube was kept at 100° C., when the height of the mercury column was 76·49 centimetres. Find from this experiment the pressure of mercury vapour at 100° C.

Answer. ·074 centimetre.

(35.) The mercury in a barometer stands at 75·2 centimetres when the temperature is 25° C. Taking the density of mercury at 0° C. at 13·596 and that of water at 25° C. equal to ·997, and the maximum pressure of aqueous vapour at 25° C. equal to 2·355 centimetres of mercury, find the corresponding height of a water barometer.

The barometric column reduced to 0° C. is $\dfrac{75 \cdot 2}{1+\dfrac{25}{5550}}$ $= 74 \cdot 863$ centimetres, and if x be the height in centimetres of the water column at 25° C. the equivalent mercury column at 0° C. is

$$\frac{x}{\cdot 997 \times 13 \cdot 596} = \frac{x}{13 \cdot 555} \text{ centimetres;}$$

And $\therefore \dfrac{x}{13 \cdot 555} + 2 \cdot 355 = 74 \cdot 863$;

Whence $x = 982 \cdot 86$ centimetres.

(36.) The internal section of a long barometer tube is 1 square centimetre, and the tube contains water vapour which, at 13° C., occupies 80 centimetres of the tube, and the top of the mercury column is 75·26 centimetres above the surface of the mercury in the cistern. The barometric pressure of the air is 76 centimetres. The cistern, which is connected to the barometer by flexible tubing, is then raised until the vapour occupies only 20 centimetres in the tube. How much will have been condensed?

The pressure of the vapour in the tube at the beginning is $76 - 75 \cdot 26 = \cdot 74$ centimetre, and therefore the quantity of aqueous vapour contained in the tube at first is

$$\frac{5}{8} \times 80 \times \cdot 001293 \times \frac{\cdot 74}{76} \times \frac{273}{286} = \cdot 006 \text{ gramme nearly.}$$

But the maximum pressure of aqueous vapour at 13° C. is 1·116 centimetre, and therefore when the vapour occupies

only 20 c.c. of the tube the quantity which would saturate this space is

$$\frac{5}{8} \times 20 \times {\cdot}001293 \times \frac{1{\cdot}116}{76} \times \frac{273}{286} = {\cdot}002 \text{ gramme nearly.}$$

$\therefore$ quantity of vapour condensed is ·006 — ·002 = ·004 gramme.

DENSITY OF VAPOURS.

(1.) It is found by experiment that a given quantity of steam at 100° C. and 76 centimetres of barometric pressure occupies about 1,700 times as much space as an equal quantity of water at 0° C.; and it is also found that dry air at 0° C. and 76 centimetres pressure occupies about 770 times as much room as an equal quantity of water at 0° C. From these data calculate the ratio of the density of steam to that of air at 100° C. and 76 centimetres of pressure.

For an increase of temperature of 1° C. the expansion of water is $\frac{1}{273}$rd of its bulk at the lower temperature, and therefore

$$\frac{\text{Density of air at } 100^\circ \text{ C.}}{\text{Density of air at } 0^\circ \text{ C.}} = \frac{1}{1 + \frac{100}{273}} = \frac{273}{373} \quad . \quad . \quad (1)$$

But by the question

$$\frac{\text{Density of steam at } 100^\circ \text{ C.}}{\text{Density of water at } 0^\circ \text{ C.}} = \frac{1}{1700} \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{\text{Density of air at } 0^\circ \text{ C. and } 76 \text{ centimetres pressure}}{\text{Density of water at } 0^\circ \text{ C.}} = \frac{1}{770} \quad . \quad . \quad (3)$$

Combining the relations (1), (2), and (3) we find that

$$\frac{\text{Density of steam at } 100^\circ \text{ C. and 76 centimetres pressure}}{\text{Density of air at } 100^\circ \text{ C. and 76 centimetres pressure}} =$$

$$= \frac{770}{1700} \times \frac{373}{273} = \cdot 62 \text{ nearly.}$$

N.B. In practice it is always assumed that the density of water vapour is always $\frac{5}{8}$ths that of dry air under the same conditions of temperature and pressure.

(2.) If steam which is still in contact with water be heated up to 130° C. its pressure becomes equal to that of 203 centimetres of mercury. What is the density of saturated steam at 130° C.?

$$\text{The density required} = \frac{5}{8} \times \cdot 001293 \times \frac{203}{76} \times \frac{273}{403}$$

$$= \cdot 0014622 \text{ gramme.}$$

(3.) What is the density of saturated aqueous vapour at 40° C., the maximum pressure at that temperature being equal to 549 centimetres of mercury?

Answer. ·00005 gramme approximately.

(4.) What is the mass of a cubic metre of steam at 100° C. and 76 centimetres pressure? *Answer.* 591·47 grammes.

(5.) Find the quantity of aqueous vapour in a cubic metre which is saturated at 25° C., the maximum pressure at that temperature being 2·36 centimetres.

Answer. 22·989 grammes.

(6.) Find the quantity of saturated steam in a cubic metre at 120° C., the maximum pressure at that temperature being 149·3 centimetres.

Answer. 1101·54 grammes nearly.

(7.) A cubic centimetre of water at 100° C. and 76 centimetres of barometric pressure is converted into steam at 100° C. What space will it occupy?

At 100° C. and 76 centimetres pressure one gramme of

dry air ocupies $\frac{1}{\cdot 001293} \times \frac{373}{273}$ c.c.

and as the density of steam is always $\frac{5}{8}$ that of air under the same conditions of temperature and pressure, the space occupied by one gramme of steam at 100° C. and 76 centimetres pressure is

$$\frac{8}{5} \times \frac{1}{\cdot 001293} \times \frac{373}{273} = 1690 \cdot 7 \text{ c.c.}$$

The student will notice from this result that the common saying that 'a cubic inch of water becomes a cubic foot of steam,' is only a very rough approximation.

(8.) One litre of oxygen and two litres of hydrogen when combined produce two litres of steam at the same temperature and pressure. Deduce from this the density of steam with respect to air.

Referring to Examples 11, 12, 13, p. 68, we find that the quantity of the mixed gases is 1·6092 gramme and that the quantity of air in 2 litres is 2·586 grammes.

$$\therefore \frac{\text{Density of steam}}{\text{Density of air}} = \frac{1 \cdot 6092}{2 \cdot 586} = \cdot 62 \text{ nearly.}$$

(9.) Find the vapour density of bisulphide of carbon, taking that of hydrogen as unity, from the following experimental data.

Capacity of the flask at 0° C. . .	= 625 c.c.
At 18° C. and 76·3 centimetres pressure the flask filled with dry air weighed	107·13 grammes.
At 90° C. and 76·2 centimetres pressure the flask filled with the vapour weighed	107·98 ,,

The coefficient of expansion of the glass $= \frac{1}{38700}$.

Solution.—The capacity of the flask at 18° C. =

$$625 \times \left(1 + \frac{18}{38700}\right) = 625 \cdot 29 \text{ c.c.}$$

∴ the quantity of air contained in the flask at 18° C. and 76·3 centimetres barometric pressure is

$$625{\cdot}29 \times {\cdot}001293 \times \frac{76{\cdot}3}{76} \times \frac{273}{291} = {\cdot}7615 \text{ gramme.}$$

∴ mass of the empty flask $= 107{\cdot}13 - {\cdot}7615 = 106{\cdot}3685$ grammes.

∴ the quantity of bisulphide of carbon vapour which fills the flask at 90° C. and 76·2 centimetres pressure is

$$107{\cdot}983 - 106{\cdot}3685 = 1{\cdot}6145 \text{ grammes.}$$

But the capacity of the flask at 90° C. is

$$625\left(1 + \frac{90}{38700}\right) = 626{\cdot}45 \text{ c.c.}$$

∴ the quantity of hydrogen which would fill the flask at 90° C. and 76·2 centimetres pressure is

$$626{\cdot}45 \times {\cdot}0000896 \times \frac{76{\cdot}2}{76} \times \frac{273}{363} = {\cdot}0423 \text{ gramme.}$$

∴ density of bisulphide of carbon vapour with respect to that of hydrogen is $\frac{1{\cdot}6145}{{\cdot}0423} = 38{\cdot}168.$

(10.) What would be the relative density (air = 1) of the bisulphide of carbon vapour?

Answer. $\frac{1{\cdot}6145}{{\cdot}6108} = 2{\cdot}643.$

(11.) Find the density of alcohol vapour with reference to hydrogen from the following experimental data.

Capacity of the flask at 0° C. . .	= 520 c.c.
Weight of flask filled with dry air at 12° C. and 75·2 centimetres pressure	= 75·771 grammes.
Weight of flask filled with alcohol vapour and sealed at 168° C. and 75·8 centimetres pressure . .	= 75·8 grammes.

Coefficient of expansion of the flask $= \frac{1}{38700}$

Answer. 2·375.

(12.) Find the absolute density of camphor vapour from the following data obtained experimentally by Dumas.

Temperature of the vapour when the flask was hermetically sealed	$= 244°$ C.
Temperature of the air	$= 13{\cdot}5°$ C.
Barometric pressure	$= 74{\cdot}2$ centimetres.
Increase in weight of flask. . .	$= 0{\cdot}708$ gramme.
Capacity of flask at $13{\cdot}5°$ C. . .	$= 295$ c.c.

Answer. ·0023857 gramme.

(13.) Find the density of camphor vapour at 244° C. with respect to hydrogen at the same temperature and pressure.

Answer. 51·648.

(14.) What will be the density of camphor vapour with respect to air at the same temperature and pressure?

Answer. 3·579.

(15.) Determine the absolute density of ether vapour from the following experimental data.

Weight of flask full of dry air at 12° C. and 76·2 centimetres pressure	$= 68{\cdot}3$ grammes.
Weight of flask filled with ether vapour and sealed at 65° C. and 76·1 centimetres pressure . .	$= 69{\cdot}114$,,
Capacity of the glass flask at 0° C.	$= 560$ c.c.
Coefficient of expansion	$= \frac{1}{38536}$

Answer. ·0026912 gramme.

(16.) Find the density of ether vapour with respect to air at the same temperature and pressure as in the last example. *Answer.* 2·5734.

(17.) Find the absolute density of the vapour of iodine from the following experimental data obtained by Dumas's method.

Weight of empty flask	=	45·5 grammes.
Weight of flask filled with iodine vapour and sealed at 200° C. and 76 centimetres pressure . . .	=	47·463 ,,
Capacity of the flask at 0° C.	=	300 c.c.
Coefficient of expansion	=	$\frac{1}{38700}$

Answer. ·0065097 gramme.

(18.) What would be the density with reference to that of air at the same temperature and pressure?

Answer. 8·7244.

(19.) What would have been the density, taking that of hydrogen at the same temperature and pressure as unity?

Answer. 125·88.

(20.) Calculate the value of the vapour density of a substance with respect to that of air as unity from the following data, which were obtained experimentally by the method of Gay Lussac.

Quantity of the substance in the small glass bulb	=	·447 gramme.
Observed volume of its vapour . .	=	120 c.c.
Corrected height of barometer . .	=	76·2 centimetres.
Difference of level of mercury inside and outside the tube .	=	6 ,,
Height of oil column	=	17 ,,
Temperature of oil-bath	=	200° C.
Density of the oil	=	·808.

The pressure of the oil column when expressed in terms of the equivalent column of mercury at 0° C. is

$$\frac{17 \times \cdot 808}{13 \cdot 596} = 10 \cdot 1 \text{ centimetres.}$$

And the mercury column in the tube, which is at 200° C. when corrected for temperature, is

$$\frac{6}{1 + \frac{200}{5550}} = 5 \cdot 79 \text{ centimetres.}$$

$\therefore$ pressure of the vapour $= 76.2 + 10.1 - 5.79 = 80.51$ centimetres.

But at this pressure and temperature the quantity of air which would occupy 120 c.c. is

$$120 \times .001293 \times \frac{80.51}{76} \times \frac{273}{473} = .0949 \text{ gramme nearly.}$$

$$\therefore \text{ Density of the vapour (air} = 1) \text{ is } \frac{.447}{.0949} = 4.71.$$

N.B. The elastic force of mercury vapour at 200° C. is 1·99 centimetre, and therefore a correction should be applied for this which would make the pressure of the vapour

$$80.51 - 1.99 = 78.52 \text{ centimetres,}$$

and this would make the vapour density with reference to that of air as unity equal to

4·83 nearly.

(21.) A wide glass tube which was graduated at 0° C. was filled with mercury and then inverted over a mercury bath. One gramme of ether was then passed up into the Torricellian vacuum and the tube was surrounded by a water bath according to the method of Gay Lussac, and the temperature was raised to 85° C. It was then found that the vapour occupied 487·64 c.c. according to the graduation of the tube, and that the difference of level of the mercury in the tube and in the bath was 14·3 centimetres. The corrected barometric height was 75·4 centimetres. Find the density of ether vapour with respect to air.

Since the glass was correctly graduated at 0° C. the true volume of the vapour at 85° C. is

$$487.64\left(1 + \frac{85}{38700}\right) = 488.71 \text{ c.c.}$$

and then, proceeding as in Example 20, we find that the relative density (air $= 1$) is 2·57.

(22.) Find the density ($H = 1$) of the vapour of ether from the following data, which were obtained by Hofmann's modification of Gay Lussac's method.

Quantity of ether taken . . .	=	·273 gramme.
Space occupied by its vapour . .	=	150 c.c.
Temperature of the vapour . .	=	100° C.
Height of mercury in the tube .	=	18·32 centimetres.
Barometric height reduced to 0° C.	=	75 „

The principle of Hofmann's method is precisely the same as in that of Gay Lussac, so that, proceeding as in the solution of Example 20, we find that

the density of ether vapour ($H=1$) is $\dfrac{\cdot 273}{\cdot 00738} = 37$ nearly.

(23.) What would have been the density, taking that of air as unity? *Answer.* 2·565 nearly.

N.B. The pressure of mercury vapour at 100° C. may be neglected, as it is only ·075 of a centimetre.

(24.) Find the relative density (air = 1) of the vapour of bisulphide of carbon from the following data of an experiment according to Hofmann's method.

Quantity of bisulphide of carbon used	=	·074 gramme.
Space in tube occupied by the vapour	=	68·4 c.c.
Temperature of the vapour . . .	=	20° C.
Barometric pressure reduced to 0° C.	=	74 centimetres.
Height of mercury column in tube	=	45·7 „

Answer. 2·39 nearly.

(25.) What would be the relative density taking that of hydrogen as unity? *Answer.* 34·6.

N.B. This was an impure sample, and probably contained a little water.

(26.) Find the density of chloroform vapour (air = 1) from the following data of an experiment according to Hofmann's method.

Quantity of chloroform taken . .	=	·096 gramme.
Space in tube occupied by its vapour	=	60 c.c.
Temperature of the vapour . . .	=	30° C.
Barometric pressure reduced to 0° C.	=	75·2 centimetres.
Height of mercury column in tube .	=	50·67 ,,

Answer. 4·21 nearly.

(27.) What is the absolute density of the chloroform vapour, *i.e.* the mass of one cubic centimetre, at the above temperature and pressure? *Answer.* ·0016 gramme.

(28.) What is the relative density taking that of hydrogen as unity? *Answer.* 60·74 nearly.

Note.—The student should notice that Gay Lussac's method is the reverse of Dumas's. In the latter we find the mass of a known bulk of vapour, while in the former we find the bulk of a given mass of vapour. Gay Lussac's method is only available for substances which are easily volatilised, whereas Dumas's method, though more complicated, is available for all.

LATENT HEAT OF VAPOUR.

(1.) Some water was heated under pressure (in a Papin's digester) to the temperature of 207° C. and the valve was then opened, when part of the water rushed out in the form of steam, and the temperature of the remainder sank to 100° C. It was then found that one-fifth of the water had escaped as steam. Deduce from this experiment the value of the latent heat of steam.

Latent heat is the quantity of heat which must be communicated to a body in a given state in order to convert it into another state without changing its temperature.

The *unit quantity* of heat is that quantity which, if applied to unit of mass of water at some standard temperature, will raise that water one degree in temperature. The

standard temperature is generally 4° C., *i.e.* that at which water has its maximum density.

Let x be the number of units of heat or the number of grammes of water which would be heated from 4° C. to 5° C. by the heat required to convert one gramme of water at 100° C. into steam at 100° C. Then x is the latent heat of steam at 100° C.

Also let m be the total quantity of water and steam in grammes.

Since the temperature of the whole mass falls from 207° to 100° C., the quantity of heat liberated is

$$107 \times m \text{ heat units};$$

and in the conversion of $\frac{m}{5}$ grammes of water at 100° C. into steam at 100° C. the quantity of heat absorbed is

$$\frac{m}{5} \times x \text{ heat units.}$$

Assuming that during this process no heat is lost to or gained from external objects, these two quantities of heat must be equal.

$$\therefore \frac{m}{5} \times x = 107 \times m;$$

$$\therefore x = 107 \times 5 = 535 \text{ units.}$$

(2.) A shallow vessel containing some water at 10° C. was placed over a burner, and in four minutes its temperature rose to 100° C. and it began to boil, and in 24 minutes more the whole had boiled away. Calculate from these data the latent heat of steam.

Let m be the mumber of grammes of water; then in one minute it absorbed $\frac{m \times 90}{4} = m \times 22{\cdot}5$ heat units.

And $\therefore$ it absorbed $m \times 22{\cdot}5 \times 24 = m \times 540$ heat units in 24 minutes.

But this was the quantity of heat which was required to

turn m grammes of water at 100° C. into steam at 100° C., and therefore the latent heat of steam is 540 heat units.

(3.) A small quantity of water at 15·5° C. is placed in an evaporating dish, which is covered by a glass plate which has a small hole in it. The flame of a gas-burner causes the water to begin boiling in 3 minutes and 20 seconds, and the whole of the water is evaporated after 21 minutes more have elapsed. Find from these data the latent heat of steam. *Answer.* 532 heat units nearly.

(4.) A Papin's digester contains 5 kilogrammes of water at 150° C. When the valve is opened a quantity of the water is immediately converted into steam, and the temperature of the rest of the water falls to 100° C. Assuming that the latent heat of steam is 537 units, find the quantity of steam produced.

Let x be the number of grammes of steam produced; then, as the final temperature of the steam and water is 100° C., the quantity of heat yielded up by the mass as its temperature falls is

$$5000 \times 50 = 250000 \text{ heat units.}$$

But this heat has been absorbed in converting x grammes of water at 100° C. into steam at 100° C.; and therefore, assuming that there was no loss or gain of heat from without,

$$x \times 537 = 250000;$$

$$\therefore x = \frac{250000}{537} = 465\cdot55 \text{ grammes.}$$

(5.) A Marcet's boiler contains a kilogramme of water which is at the temperature of 123° C. If the stopcock be opened, what quantity of steam will be produced?

Answer. 42·83 grammes.

(6.) By keeping some water perfectly still in a vessel lined with shell-lac it was raised to 105° C., and then ebullition took place in bursts, each burst causing the temperature

to fall to 100° C. What fraction of the water was converted into steam at each burst?

Let x = number of grammes of steam.
m = „ „ water and steam.

Just before ebullition the whole mass contained $105\,m$ heat units, and just after ebullition the water contained $(m - x) \times 100$ heat units, while the steam contained $(100 + 536)\,x = 636\,x$ heat units.

And if there has been no loss or gain of heat from without, the number of heat units in the water just *before* ebullition must be equal to the total number in the water and steam just *after* ebullition, and

$$\therefore\ (m - x) \times 100 + 636\,x = 105\,m.$$

$$\therefore\ 536\,x = 5\,m.$$

$$\therefore\ \frac{x}{m} = \frac{5}{536} = \frac{1}{107} \text{ nearly.}$$

(7.) On another occasion the water in a glass vessel was raised to 102° C. just before ebullition commenced. What fraction of the whole quantity of water was turned into steam at each burst? *Answer.* $\frac{1}{268}$.

N.B. In both these cases the barometric pressure was 76 centimetres of mercury.

(8.) If the latent heat of steam is 536 when the units are the gramme and the Centigrade degree, what will be the number representing the latent heat of steam when the pound avoirdupois and one degree Fahrenheit are the units?

Since one gramme of water at 100° C. in becoming steam at 100° C. absorbs 536 water-gramme-degrees Centigrade, therefore one pound of water will absorb 536 water-pound-degrees Centigrade, or $\frac{9}{5} \times 536 = 964{\cdot}8$ water-pound-degrees Fahrenheit.

$\therefore$ latent heat of steam in terms of the new units is $964{\cdot}8$.

(9.) Having given that one pound is equal to 453·59 grammes, find the ratio which the quantity of heat represented by one water-pound-degree Fahrenheit bears to the quantity represented by one water-gramme-degree Centigrade. *Answer.* 252 very nearly.

(10.) How much steam at 100° C. and 76 centimetres pressure must be passed into 600 kilogrammes of water initially at 15° C. so as to raise the temperature of the whole to 70° C.? It is assumed that during the process 40 per cent. of the total heat is lost by radiation and communication to surrounding objects, and the latent heat of steam is 537.

The heat gained by the 600 kilogrammes of water as the temperature rises from 15° C. to 70° C. is

$$600 \times 55 = 33000 \text{ kilogramme units.}$$

And if x be the number of kilogrammes of steam required, the quantity of heat given up in condensing and cooling to 70° C. is

$$x\,(537 + 30) = x \times 567 \text{ kilogramme units.}$$

And since the heat gained by the water is only 60 per cent. of the heat given up by the steam, we have

$$33000 = \frac{3}{5} \times x \times 567.$$

$$\therefore x = \frac{33000 \times 5}{3 \times 567} = 97 \text{ kilogrammes nearly.}$$

(11.) How much steam at 100° C. is required in order to raise the temperature of 620 pounds of water from 0° C. to 100° C., 20 per cent. of the heat being lost by radiation and other causes? *Answer.* 144·32 pounds nearly.

(12.) What would be the latent heat of water vapour at 60° C. if Watt's law be true?

Watt came to the conclusion that the same total quantity of heat is required in order to evaporate the same mass of

water at all temperatures. But when the heating begins at $0°$ C. and evaporation takes place at $100°$ C., we know that the total heat of evaporation of a gramme of water is

$$100 + 536 = 636 \text{ units.}$$

Let x be the latent heat of steam at $60°$ C., then if the water be first heated from $0°$ C. to $60°$ C., the total heat of evaporation is $60 + x$ units, and if Watt's law holds then

$$60 + x = 636 \;\therefore\; x = 576 \text{ units.}$$

(13.) Assuming Watt's law to be true, what is the latent heat of steam at $150°$ C.? *Answer.* 486 units.

(14.) At what temperature would the latent heat of saturated steam be zero if Watt's law still holds?

Answer. $636°$ C.

N.B. In practice it is found that Watt's law holds good only for temperatures a little above or below $100°$ C.

(15.) Regnault found that the total heat of evaporation of water may be represented by the equation

$$Q = 606{\cdot}5 + 0{\cdot}305t,$$

where t is the temperature of the water at which the steam is produced. Find by Regnault's formula the latent heat of steam at $60°$ C.

Let x be the latent heat of steam at $60°$ C., then

$$x + 60 = Q = 606{\cdot}5 + {\cdot}305 \times 60 = 624{\cdot}8.$$
$$\therefore x = 564{\cdot}8 \text{ units.}$$

(16.) What would be the latent heat of water vapour at $0°$ C., according to Regnault's formula?

Answer. $606{\cdot}5$ units.

(17.) At what temperature would the latent heat of steam be zero according to Regnault's formula?

Let $x°$ C. be the required temperature, then by the formula in Example 15, we have

$$x = 606{\cdot}5 + {\cdot}305 \times x.$$
$$\therefore x = 872{\cdot}66° \text{ C.}$$

Note.—At this temperature no heat would be required to

convert the liquid into vapour, so that if Regnault's formula still held good the temperature of 872·7° C. would be the 'critical temperature' of water, that is to say, the temperature above which the properties of the liquid are not separated from those of the vapour by any apparent distinction between them.

(18.) The steam in a boiler is at 120° C. Find the total heat per gramme of steam and also the latent heat.

By Regnault's formula we have for the total heat

$$Q = 606{\cdot}5 + {\cdot}305 \times 120 = 643{\cdot}1 \text{ gramme units.}$$

$$\text{The latent heat} = 643{\cdot}1 - 120 = 523{\cdot}1 \text{ gramme units.}$$

(19.) Find the total heat and the latent heat per gramme of the steam in a boiler which is at 160° C.

Answers. Total heat = 655·3 gramme units.
Latent „ = 495·3 „ „

(20.) When the temperature of the steam in a boiler is equal to 180° C., what is the total heat and latent heat per gramme of steam?

Answers. Total heat = 661·4 gramme units.
Latent „ = 481·4 „ „

(21.) One pound of steam at 100° C. is passed into a vessel containing 10 pounds of water at 15° C. and is there condensed. What will be the temperature of the mixture?

Let $t°$ C. be the resulting temperature of the mixture, then the total heat in the steam and water *before* mixture is

$$(100 + 537) + 10 \times 15 = 787 \text{ pound units,}$$

and the total heat *after* mixture is

$$(1 + 10) \times t = 11t \text{ pound units,}$$

and if there is no loss or gain of heat from without these quantities of heat must be equal, therefore

$$11t = 787 \quad \therefore t = 71\tfrac{6}{11}° \text{ C.}$$

(22.) One pound of steam at 100° C. is condensed by

pounds of water at 0° C. What is the temperature of the resulting mixture? *Answer.* 91° C.

(23.) The latent heat of steam at 100° C. being 537, how much steam at this temperature must be condensed in 2 gallons of water at 6° C. that the temperature of the resulting mixture may be 45° C.?

One gallon = 4541 c.c.

Density of water at 6° C. = ·99997.

$\therefore$ the quantity of condensing water = $2 \times 4541 \times {\cdot}99997$
$= 9081{\cdot}7$ grammes.

Let x be the number of grammes of steam which are required, then, assuming that there is no loss or gain of heat from without, we must have

$$\left.\begin{array}{l}\text{Total heat of steam and water} \\ \text{before mixture . . .}\end{array}\right\} = \left\{\begin{array}{l}\text{Total heat of water in} \\ \text{resulting mixture.}\end{array}\right.$$

$$\therefore x \times 637 + 9081{\cdot}7 \times 6 = (x + 9081{\cdot}7) \times 45,$$

$$\text{whence } x = \frac{9081{\cdot}7 \times 39}{592} = 598{\cdot}29 \text{ grammes.}$$

(24.) A bath contains 60 gallons of water initially at 15° C. What quantity of steam at 100° C. must be led into it so as to bring its temperature up to 30° C.?

Answer. 6·727 kilogrammes, nearly.

(25.) The temperature of the injection water of a condensing engine is 15° C. The steam enters the condenser at the temperature of 100° C., and the water pumped out of the condenser is at 40° C. What quantity of injection water must be supplied for each pound of steam that enters the condenser?

Let x be the number of pounds of water required, then the total quantity of heat in the steam and water *before* condensation is $(637 + 15x)$ units, and the total quantity of heat in the condensed steam and the injection water *after* condensation is $(1 + x)\,40$ units, and since these must be equal, we have

$$(1 + x)\,40 = 637 + 15x.$$

$$\therefore x = 23{\cdot}88 \text{ pounds.}$$

(26.) How many pounds of water at 15° C. must be mixed with 20 lbs. of steam at 100° C. in order to produce water at 45° C.? *Answer.* $394\frac{2}{3}$ lbs.

(27.) Assuming that the area of the Valley of the Mississippi is 982,000 square miles, and that the mean annual rainfall is 40 inches, how many tons of coal would have to be burnt to produce an amount of heat equal to that which is annually set free among the clouds by the condensation? Each pound of coal is assumed to develop sufficient heat to raise the temperature of 8,080 lbs. of water 1° C.

Assuming that a cubic foot of water contains 1,000 ounces, the quantity of rain is

$$\text{M} = \frac{982000 \times (1760 \times 3)^2 \times 40 \times 1000}{12 \times 16} \text{ pounds,}$$

and the quantity of heat, in pound degrees, which is required to condense this is M × 537 units.

$$\therefore \text{ tons of coal required} = \frac{\text{M} \times 537}{8080 \times 20 \times 112}$$

$$= \frac{982000 \times (5280)^2 \times 40 \times 1000 \times 537}{12 \times 16 \times 8080 \times 20 \times 112}$$

$$= 1{\cdot}6922 \times 10^{11} \text{ tons nearly.}$$

(28.) What quantity of this coal would be required in order to boil away entirely a ton of water which was initially at 20° C.? *Answer.* $171\frac{5}{101}$ lbs.

(29.) The worm-tub of a still contained 50 kilogrammes of water which was at 8° C. when it was introduced, and it was replaced by fresh water as soon as it reached 25° C. The steam entered the worm-tube at 100° C., and the condensed water issued from the worm at 25° C. How many times would the water in the tub have to be renewed in order to get $11\frac{1}{9}$ kilogrammes of distilled water?

Let x be the number of times that the water in the tub had to be renewed.

Then the heat gained by the water in tub

$$= x \times 50 \times (25 - 8) = 850\,x \text{ units,}$$

and the heat lost by the steam

$$= \frac{100}{9}(537 + 75) = \frac{61200}{9} \text{ units.}$$

But the heat gained by the water = heat lost by the steam.

$$\therefore 850\,x = \frac{61200}{9}$$

$$\therefore x = \frac{61200}{9 \times 850} = 8.$$

(30.) A small glass beaker containing 50 grammes of ether at 35° C. is placed in another beaker containing 100 grammes of water at 60° C. The boiling point of ether being 35° C., and the latent heat of its vapour 90 units, how much of the ether will evaporate, assuming that all the heat lost by the water goes to the ether?

In cooling through 25° C. the 100 grammes of water part with 2,500 heat units.

Also x grammes of ether at 35° in changing into vapour absorb $x \times 90$ heat units, and since the heat lost by the water = heat gained by the ether,

$$2500 = 90\,x.$$

$$\therefore x = 27\tfrac{7}{9} \text{ grammes.}$$

(31.) Into the same two beakers were placed 50 grammes of absolute alcohol at 78·5° C., and into the outer beaker 100 grammes of water at 92° C. The boiling point of the alcohol being 78·5° C., and the latent heat of its vapour 193 units, find how much of the alcohol will evaporate.

Answer. 6·994 grammes.

LATENT HEAT OF WATER.

(1.) Two thin glass flasks were suspended in a room in which the air was at $8\cdot5^\circ$ C. One of the flasks contained water at 0° C., and the other contained the same quantity of ice at 0° C. At the end of half an hour the temperature of the water had risen 4° C., and at the end of $10\frac{1}{2}$ hours the ice had all melted and acquired the same temperature. Find the number of heat units required to melt a pound of ice at 0° C.

Let m be the number of pounds of water or ice, then the water absorbs $4\,m$ units of heat in half an hour, and $\therefore$ $\frac{21}{2} \times 8\,m = 84\,m$ units in $10\frac{1}{2}$ hours.

Also the heat absorbed by the ice in $10\frac{1}{2}$ hours is m $(x + 4)$ units, and as both the flasks were in the same room we may assume that they received equal increments of heat in equal times.

$$\therefore\ m(x + 4) = 84\,m \quad \therefore\ x = 80 \text{ units.}$$

(2.) A quantity of ice at 0° C. was exposed to a constant source of heat, and it was found that it took eight minutes to reduce the mass of ice to water at 0° C. It took ten minutes more to raise the temperature of the water to 100° C., and thenceforward the temperature remained constant till, after a further interval of fifty-four minutes, the whole of the water had been converted into steam. Deduce from this experiment the latent heat of water, and also that of steam.

Answers. Latent heat of water $=$ 80.
" " steam $=$ 540.

N.B. Unless otherwise stated the latent heat of water is to be taken as 80 units in the following examples.

(3.) Three pounds of crushed ice at 0° C. were immersed in 7 pounds of water at 100° C., and the final temperature of the mixture was $46\cdot2^\circ$ C. Find from these data the

quantity of heat which is absorbed in the liquefaction of a pound of ice.

Let x be the number of units of heat required.

The heat lost by the water $= 7 \times (100 - 46{\cdot}2) = 376{\cdot}6$ units.

The heat gained by the ice $= 3 \times (x + 46{\cdot}2)$ units.

And assuming that there is no loss or gain of heat from without, these quantities of heat must be equal.

$$\therefore\ 3\,(x + 46{\cdot}2) = 376{\cdot}6.$$
$$\therefore\ x = 79\tfrac{1}{3} \text{ units.}$$

(4.) A kilogramme of crushed ice at 0° C. was thrown into 2 kilogrammes of water at 25° C. Assuming that there is no gain of heat from outside, what quantity of ice will have melted when the water has just cooled down to 0° C.?

Let x = number of grammes of ice melted, then
the water loses $25 \times 2000 = 50000$ units,
the ice gains $80\,x$ units, and these must be equal.
$\therefore\ 80\,x = 50000 \quad \therefore\ x = 625$ grammes.

(5.) Find the temperature of the water obtained by pouring 5 pounds of boiling water over 3 pounds of ice at 0° C.

Let t° C. be the required temperature, then
the heat lost by the water $= 5\,(100 - t)$ units.
,, gained by the ice $= 3\,(80 + t)$,,
and $\therefore$ $3\,(80 + t) = 5\,(100 - t)$
$\therefore\ t = 32{\cdot}5°$ C.

(6.) What will be the final temperature when 6 pounds of crushed ice at 0° C. are mixed with 18 pounds of water at 90° C.? *Answer.* 47·5° C.

(7.) How many thermal units will be required in order to convert 6 pounds of ice at 0° C. into steam at 100° C.?

Answer. $6\,(80 + 100 + 537) = 4{,}302$ units.

(8.) Some water is at the temperature of 50° C., and

4 pounds of crushed ice at 0° C. were thrown in. When all the ice had melted the temperature of the mixture was 30° C. What was the original quantity of water?

Let x be the original number of pounds of water, then
the heat lost by the water $= x \times 20$ units,
" gained by the ice $= 4\ (80 + 30) = 440$ units,
and as before $x \times 20 = 440 \quad \therefore\ x = 22$ pounds.

(9.) What quantity of water at 15° C. will be required to melt 10 pounds of ice at 0° C., so that the resulting mixture should be at 5° C.? *Answer.* 85 pounds.

(10.) How many pounds of steam at 100° C. will just melt 20 pounds of ice at 0° C., the latent heat of steam being 537 units? *Answer.* 2·51 pounds nearly.

(11.) How many pounds of crushed ice are required in order to condense and then cool down to 0° C. 10 pounds of steam at 100° C.? *Answer.* $79\frac{5}{8}$ pounds.

(12.) What quantity of ice would have been required for the final temperature of the mixture to have been 15° C.? *Answer.* 65·47 pounds.

(13.) If a quantity of water be kept perfectly still it can be cooled down to $-10°$ C. or even lower without freezing, but if it be then disturbed a portion of it at once becomes solid. What proportion does this part bear to the whole?

Let m be the number of grammes of water originally.
m' " " " " that solidify.

The $(m - m')$ grammes of water when the temperature has risen from $-10°$ C. to 0° C. have absorbed $10\ (m - m')$ heat units.

The m' grammes of water in solidifying to ice at 0° C. have absorbed $10\ m'$ units and given out $80\ m'$ units, so that on the whole they have given out $70\ m'$ units.

$$\therefore\ 70\ m' = 10\ (m - m')$$

$$\therefore\ \frac{m'}{m} = \frac{1}{8}.$$

(14.) Fifty pounds of water were carefully cooled down to $-8°$ C. without solidifying. On slightly agitating the water a portion solidified and the temperature of the remainder rose to $0°$ C. How much ice was formed?

Answer. 5 pounds.

(15.) Assuming that the density of snow at $0°$ C. is ·52, and that 3 inches of snow are on the ground, how many inches of rain at $10°$ C. must fall so as just to melt all the snow?

The quantity of snow per square foot of ground

$$= \frac{·52 \times 1000}{4 \times 16} \text{ pounds,}$$

$$= 8·125 \text{ pounds,}$$

and if x be the number of inches of rain required the number of pounds per square foot will be

$$\frac{x}{12} \times \frac{1000}{16} \text{ pounds,}$$

but each pound of snow absorbs 80 units of heat.

$$\therefore\ 10\,x \times \frac{1000}{12 \times 16} = 8·125 \times 80.$$

$$\therefore\ x = 12·48 \text{ inches.}$$

(16.) If the layer of snow were 2 centimetres thick and of the same density as before, its temperature being $0°$ C., how many centimetres of rain at $12·5°$ C. must fall so as just to melt the snow? *Answer.* 6·65 centimetres.

WATER CALORIMETERS.

(1.) Find the specific heat of iron from the observation that when 96 grammes of iron at $100°$ C. are immersed in 158·4 grammes of water at $10°$ C., after the temperatures of the water and of the iron have become equalised their common temperature is $15·8°$ C.

Let m_1 be the number of grammes of iron.
m_2 " " " water.
t_1 be the original temperature of the iron.
t_2 " " " water.
T be the final " " mixture.
x " required specific heat of the iron.

As the temperature of the water rises from t_2 to T the water gains m_2 (T $-t_2$) heat units, and as the temperature of the iron falls from t_1 to T the iron loses $m_1 x (t_1-\text{T})$ units. If we assume that there is no loss or gain of heat from without, these quantities of heat must be equal, and therefore

$$m_1 x (t_1-\text{T}) = m_2 (\text{T} - t_2)$$

$$\therefore \; x = \frac{m_2}{m_1} \times \frac{\text{T}-t_2}{t_1-\text{T}}$$

$$= \frac{158{\cdot}4}{96} \times \frac{5{\cdot}8}{84{\cdot}2} = {\cdot}114 \text{ nearly.}$$

(2.) When a kilogramme of mercury at 100° C. was mixed with a kilogramme of water at 7° C., the final temperature of the mixture was 10° C. What was the specific heat of the mercury? *Answer.* $\frac{1}{30}$.

(3.) How many heat units are necessary to raise the temperature of 360 grammes of mercury from 20° C. to 60° C.?

If m be the number of grammes of the substance, s its specific heat, and θ the change of temperature, the number of heat units required is

$$\text{Q} = m.s.\,\theta$$

$$= 360 \times \frac{1}{30} \times 40 = 480 \text{ gramme units.}$$

(4.) How many heat units are required to raise 80 kilogrammes of iron from 5° C. to 155° C.?

Answer. 1,368 kilogramme units.

(5.) How many thermal units are required to raise 11

kilogrammes of mercury from 4° C. to 32° C., the capacity for heat of mercury being ·0319?

Answer. 9825·2 gramme-Centigrade units.

(6.) A calorimeter contains 400 grammes of water at 10° C., and after 24·5 grammes of water at 80° C. have been added the final temperature of the mixture is 14° C. Find the water-equivalent of this calorimeter.

If m be the mass of the calorimeter expressed in grammes and s represent the specific heat of the material of which the calorimeter is made, then the product $m \times s$ is called the *water-equivalent* of the calorimeter, and is equal to the number of grammes of water which would absorb the same quantity of heat as the calorimeter actually does while its temperature rises one degree.

Let w be the water-equivalent of this calorimeter, then since the calorimeter and the water contained in it are at the same temperature, namely, 10° C., they are together equivalent to $(w + 400)$ grammes of water at 10° C. But the heat lost by the added water must be equal to the heat gained by the calorimeter and the water which it contained originally, therefore

$$24{\cdot}5 \times 66 = 4\,(w + 400)$$

$$\therefore\ w = \frac{17}{4} = 4{\cdot}25 \text{ grammes.}$$

(7.) This calorimeter was of brass and weighed 50 grammes. What should we infer from this to be the value of the specific heat of brass?

$$\text{Since } w = m \times s$$

$$\therefore\ s = \frac{w}{m} = \frac{4{\cdot}25}{50} = {\cdot}085.$$

(8.) A small brass calorimeter weighing 1·2 grammes contained 15 grammes of water at 10° C. A thermometer was heated up to 50° C. and then its bulb was plunged into the calorimeter and the final temperature was 13° C. What

was the water-equivalent of this thermometer up to the freezing point?

By the method of Example 6 we find $w = 1{\cdot}225$ gramme.

(9.) A thermometer was heated to 60° C. and was then plunged into 30 grammes of water at 12° C. and the final temperature was 13° C. What was the water-equivalent of this thermometer?

Answer. $w = \frac{30}{47}$ gramme.

(10.) What was the water-equivalent of a thermometer which when heated to 75° C. and plunged into 27·5 grammes of water at 11° C. produced a final temperature of 13° C.?

Answer. $\frac{55}{62}$ of a gramme.

Note.—The correction for the water-equivalent of the thermometer is never very accurate because the thermometer is only partially immersed in the liquid. In practice the mass of the thermometer used in such experiments is usually very small, so that the error arising from not allowing for the proper portion of the thermometer actually immersed is so small that it may be neglected.

(11.) Find the specific heat of mercury from an experiment in which 80 grammes of mercury were heated to 98° C. and were then thrown into a calorimeter which, with the water contained in it, was equivalent to 112 grammes of water, the initial temperature being 10° C. The final temperature of the mixture was 12° C.

By the method of Example 1,

$$s = {\cdot}033 \text{ nearly.}$$

(12.) Fifteen grammes of lead at 100° C. are dropped into a vessel containing water at 22° C. The heat capacity of the calorimeter and of the water contained in it being equal to that of 30 grammes of water, and the final temperature of the mixture being 23·54° C., find the specific heat of lead. *Answer.* ·04028.

(13.) In one of Regnault's experiments 293·65 grammes of zinc at 99·11° C. were immersed in 462·39 grammes of water at 0° C. The zinc was contained in a brass cage of 8·48 grammes, and the calorimeter was of brass and weighed 55·14 grammes. The rise of temperature of the water was 5·22° C., and the glass of the thermometer weighed 1·27 gramme, and the mercury contained in it 7·62 grammes. The specific heat of brass being ·094, that of mercury ·033, and that of glass ·198, find the specific heat of this sample of zinc.

By the method of Example 6, the water-equivalent of the calorimeter and thermometer was

$$5·1841 + ·5029 = 5·687 \text{ grammes,}$$

and if x be the specific heat of zinc, the heat gained by the calorimeter and its original contents being equal to the heat lost by the zinc and its cage, we have

$$93·89\,(293·65\,x + 8·48 \times ·094) = 5·22 \times (5·687 + 462·39)$$
$$= 5·22 \times 468·077.$$

Solving this equation we get $x =$ ·086 nearly.

(14.) Find the mean specific heat of water between 20·5° C. and 107·7° C. from the following experiment of Regnault's.

The sheet-iron calorimeter weighed 6931·6 grammes and contained at first 99626·6 grammes of water at 11·7° C. To this he added 10059·8 grammes of water at 107·7° C. The final temperature of the mixture was 20·5° C., and it was known that the temperature of the mixture was lowered ·03° C. by the external air. The specific heat of the sheet iron was ·1138.

The water-equivalent of the calorimeter $= 6931·6 \times ·1138$ $= 788·81$ grammes.

$\therefore$ calorimeter + water $= 788·81 + 99626·6 = 100415·41$ grammes.

Let x be the mean specific heat of water between 20·5° C.

and 107·7° C., that of water up to 20·5° C. being unity; then, since the heat lost by the added water is equal to the heat gained by the calorimeter and its original contents,

$$87 \cdot 17 \times 10059 \cdot 8 \times x = 100415 \cdot 41 \times 8 \cdot 83.$$

$$\therefore x = \frac{100415 \cdot 41 \times 8 \cdot 83}{87 \cdot 17 \times 10059 \cdot 8} = 1 \cdot 0111.$$

(15.) Find the specific heat of some white marble from the following experimental data obtained by Regnault.

Quantity of marble taken .	= 130·46 grammes.
Water-equivalent of cage .	= 0·601 gramme.
Water-equivalent of calorimeter and thermometer .	= 5·7 grammes.
Quantity of water in calorimeter	= 462·45 „
Initial temperature of the marble	= 96·85° C.

ICE CALORIMETERS.

(1.) A small hole was made in a block of ice in imitation of Black's calorimeter, and a leaden bullet of 88 grammes at 100° C. was dropped in and a lid of ice put on. After a few minutes the bullet was taken out and the water produced by the melting of the ice was removed by a pipette and found to be 3·3 grammes. The hole was then enlarged, 88 grammes of boiling water were poured in, and when all the water in the hole had acquired the temperature of 0° C. it was removed with a pipette and found to be 198 grammes. Deduce from this experiment the specific heat of lead.

The quantity of ice melted by the hot water is 198 – 88 = 110 grammes.

∴ 88 grammes of water at 100° C. will melt 110 grammes of ice,

and 88 grammes of lead at 100° C. will melt 3·3 grammes of ice ;

$$\therefore \frac{\text{quantity of heat in one gramme of lead at } 100^\circ \text{ C.}}{\text{quantity of heat in one gramme of water at } 100^\circ \text{ C.}}$$

$$= \frac{3{\cdot}3}{110} = {\cdot}0\dot{3}.$$

$\therefore$ specific heat of lead = ·03.

(2.) An iron ball weighing 220 grammes and at 100° C. was placed in a hole in a block of ice and an ice lid was placed over the hole. When the ball had cooled to 0° C. it was found that 32·23 grammes of ice had been melted. Taking the latent heat of water at 80 units, find the specific heat of iron. *Answer.* ·1172.

(3.) How much snow at 0° C. must be added to 30 grammes of alcohol at 8° C. so as to reduce its temperature to 0° C., the specific heat of alcohol being ·67 ?

Let x be the number of grammes of snow required, then the heat gained by the snow being equal to the heat lost by the alcohol,

$$80\,x = 30 \times 8 \times {\cdot}67 = 160{\cdot}8.$$

$$\therefore x = \frac{160{\cdot}8}{80} = 2{\cdot}01 \text{ grammes.}$$

(4.) When 30 kilogrammes of platinum at 100° C. were placed in a Lavoisier's calorimeter and had cooled down to 0° C., it was found that 1,215 grammes of ice had been melted. Find from this experiment the specific heat of platinum. *Answer.* ·0324.

(5.) In an experiment with Lavoisier's ice calorimeter a copper ball of 3·6 kilogrammes, which had been heated to 100° C., was introduced into the calorimeter and 146 grammes of water escaped from the ice. Find the specific heat of copper. *Answer.* ·0325 nearly.

(6.) In another experiment with the same calorimeter 6 kilogrammes of iron at 80° C. just melted 604 grammes of ice. Find the specific heat of this iron.

Answer. ·1007 nearly.

(7.) A kilogramme of small pieces of lead was placed in a brass wire cage weighing 25 grammes, and the whole was then heated to 100° C. and placed in a Lavoisier's ice calorimeter. When the temperature of the lead and cage had fallen to 0° C. it was found that 39·6 grammes of ice had been melted. Assuming the specific heat of brass to be ·094, find the specific heat of lead.

Let x be the specific heat of lead, then in cooling from 100° C. to 0° C. the lead and brass part with

$$100\ \{1000\,x + 25 \times \cdot 094\}\ \text{gramme units of heat,}$$

$$\text{and the ice gains } 39\cdot 6 \times 80 \quad ,, \quad ,,$$

$$\therefore 100\ \{1000\,x + 25 \times \cdot 094\} = 39\cdot 6 \times 80,$$

$$\text{whence } x = \cdot 0293.$$

(8.) It is found by experiment that 30 grammes of copper at 100° C. are just sufficient to melt 3·45 grammes of ice at 0° C. Find from this experiment the specific heat of copper. *Answer.* ·092.

Note.—Lavoisier's calorimeter is subject to several very serious sources of error, which have caused it to be dismissed from the ranks of exact physical instruments, but in the hands of Lavoisier it furnished very good results.

(9.) Twelve grammes of a metal at 100° C. are immersed in a mixture of ice and water, and when the metal has cooled to 0° C. the bulk of the mixture is found to have diminished by 150 cubic millimetres without change of temperature. Find the specific heat of the metal.

The density of water at 0° C. is ·99987.
,, ,, ice ,, ·91674.

∴ one gramme of ice at 0° C. occupies 1·09082 c.c.
,, ,, water ,, 1·00013 ,,

∴ one gramme of ice at 0° C. in melting to water at 0° C. diminishes in bulk by 90·69 cubic millimetres.

∴ a contraction of 150 cubic millimetres corresponds to the fusion of $\frac{150}{90\cdot 69} = 1\cdot 654$ grammes of ice,

and this represents the absorption by the ice of $1{\cdot}654 \times 80 = 132{\cdot}32$ gramme units of heat.

Let x be the specific heat of the metal, then, as before, we have

$$x \times 12 \times 100 = 132{\cdot}32.$$
$$\therefore x = \quad {\cdot}11027.$$

N.B. This example illustrates the principle of Bunsen's calorimeter, which consists in measuring the quantity of ice melted by the contraction which this ice undergoes on liquefaction. The student will find a full description of the apparatus and the experiments made therewith in 'Phil. Mag.' 1871.

(10.) In one of Bunsen's series of experiments the average quantity of ice which was melted at each experiment was ·35 gramme. What was the average quantity of heat imparted to the calorimeter at each experiment?

Answer. 28 gramme units.

(11.) In Bunsen's calorimeter the change of volume of the mixture of ice and water is measured in a long horizontal graduated glass tube containing mercury. At 9° C. the thread of mercury which occupied 507·4 subdivisions of the tube weighed ·5326 gramme. What was the capacity of each subdivision of the tube?

The capacity of one subdivision

$$= \frac{1}{507{\cdot}4} \times \frac{{\cdot}5326}{13{\cdot}596} \times \frac{5559}{5550} \text{ c.c.} = 7{\cdot}733 \times 10^{-5} \text{ c.c.}$$
$$= {\cdot}07733 \text{ cubic millimetre.}$$

(12.) Through how many subdivisions of the tube would the thread of mercury recede for an absorption by the calorimeter of 1 water-gramme-degree Centigrade of heat?

By Example 9, one gramme of ice in melting contracts by 90·69 cubic millimetres, and absorbs in doing this 80 gramme units of heat.

$\therefore$ 1 gramme unit of heat corresponds to a contraction of

$$\frac{90 \cdot 69}{80} = 1 \cdot 13362 \text{ cubic millimetre.}$$

$$= \frac{1 \cdot 13362}{\cdot 07733} \text{ subdivisions.}$$

$$= 14 \cdot 65 \quad \text{"}$$

(13.) Find the quantity of melted ice which corresponds to a change of volume of one subdivision of the tube.

Answer. ·853 milligramme.

(14.) A gramme of brass at 37° C. was dropped into the Bunsen's calorimeter, and caused the end of the thread of mercury to retreat through 50 subdivisions of the tube. Deduce from this the specific heat of brass.

Let x be the specific heat of brass; then, as the heat given out by the brass is equal to the heat absorbed by the ice,

$$x \times 37 = \cdot 000853 \times 80 \times 50 = 3 \cdot 412.$$

$$\therefore x = \frac{3 \cdot 412}{37} = \cdot 0922.$$

(15.) In another experiment the test tube contained a small quantity of water at 0° C., and when ·35 of a gramme of iron at 60° C. was dropped in, the retrocession of the mercury thread was 35·2 subdivisions. Deduce from this the specific heat of iron. *Answer.* ·1144 nearly.

N.B. With the Bunsen calorimeter the quantity of the substance used need not exceed ·3 of a gramme, whereas with the Lavoisier calorimeter satisfactory determinations of specific heat can scarcely be obtained unless the quantity of the substance employed is from 10 to 40 grammes.

GENERAL CALORIMETRY.

(1.) Twenty cubic centimetres of alcohol and the same bulk of water are exposed to the same cooling atmosphere, and are found to cool down through the same interval of

temperature in 7 and 15 minutes respectively. If the density of the alcohol be ·8, what is its specific heat?

Let $x =$ specific heat of the alcohol.

$m =$ number of grammes of water.

and $\frac{8m}{10} =$ " " alcohol.

Let $\theta =$ fall of temperature in Centigrade degrees.

The quantity of heat lost by the alcohol $= \frac{8m}{10} \times x \times \theta.$

" " " water $= m \times \theta.$

And since they are both exposed to the same cooling atmosphere the quantities of heat abstracted from them must be proportional to the times, and

$$\therefore \frac{x \times \cdot 8}{1} = \frac{7}{15}.$$

$$\therefore x = \frac{7}{15} \times \frac{10}{8} = \frac{7}{12}.$$

(2.) Find the specific heat of mercury from the observation that when the same vessel is successively filled with water and with mercury, and heated to the same temperature, the water and the mercury cool through the same number of degrees in 240 and 108 seconds respectively. The density of mercury is assumed to be constant and equal to 13·6.

Answer. ·033.

(3.) Find the latent heat of fusion of phosphorus from the following data:—

In a calorimeter containing some hot water 80·5 grammes of phosphorus were melted, and the whole was then allowed to cool quietly. The phosphorus remained liquid until its temperature had fallen considerably below that corresponding to its usual temperature of solidification. Suddenly it solidified, and the temperature of the whole increased by 5·7° C. The water equivalent of the calorimeter and water was 58·75

grammes, and the specific heat of phosphorus, whether solid or liquid, is approximately ·2.

Let x be the latent heat of fusion of phosphorus; then, since the heat given up by the phosphorus in solidifying is equal to the heat gained by the calorimeter, the water, and phosphorus,

$$80{\cdot}5 \times x = (58{\cdot}75 + 80{\cdot}5 \times {\cdot}2)\,5{\cdot}7 = 74{\cdot}85 \times 5{\cdot}7.$$

$$\therefore\ x = \frac{74{\cdot}85 \times 5{\cdot}7}{80{\cdot}5} = 5{\cdot}31.$$

(4.) Find the latent heat of fusion of ice from the following data:—

Water equivalent of calorimeter and water	=	720·3 grammes.
Quantity of ice melted	=	113·6 „
Initial temperature of water in calorimeter	=	30° C.
Final „ „ „ „	=	15° C.

Answer. 80·11.

(5.) A sheet-iron calorimeter of 512 grammes contained 19·5 kilogrammes of water at 10·2° C., and after 198·6 grammes of steam at 100° C. had been passed into it, the temperature of the water in the calorimeter was 16·5° C. The specific heat of the calorimeter was ·11. Neglecting any loss of heat by radiation, calculate from this experiment the latent heat of steam. *Answer.* 537 nearly.

(6.) The copper worm-tub of a still, with its tube, weighs 502·5 grammes, and the tub contains 600 grammes of water. The temperature of the water rose 4·5° C. after 157·67 grammes of air had passed through the tube, the air entering at 88° C. and leaving the tube at a mean temperature of 10° C. The specific heat of copper being ·095, find from the data of this experiment the specific heat of air at constant pressure.

Let x be the specific heat of air at constant pressure; then

Heat lost by the air $= 157{\cdot}67 \times 78 \times x$ heat units.

Heat gained by the calorimeter and water

$$= (502{\cdot}5 \times {\cdot}095 + 600) \times 4{\cdot}5 = 2914{\cdot}82 \text{ units nearly.}$$

$$\therefore x = \frac{2914{\cdot}82}{157{\cdot}67 \times 78} = {\cdot}237.$$

(7.) The water equivalent of the worm-tub, spiral tube, and water was 950 grammes, and the initial temperature of the water was 4° C. After 144 grammes of carbonic oxide at 80° C. had passed through the tube the temperature of the water was 5·7° C., and the mean temperature of the gas as it issued from the tube was 35° C. Calculate from this experiment the value of the specific heat of carbonic oxide at constant pressure. *Answer.* ·249.

(8.) Calculate the latent heat of steam from the following data, which were obtained by passing a measured quantity of steam through a copper tube contained in a copper calorimeter :—

Mass of the calorimeter and copper tube	=	3728·8 grammes.
" water in calorimeter . .	=	19148 "
" steam condensed . .	=	242·42 "
Initial temperature of the water . .	=	18° C.
" " " steam . .	=	100° C.
Final " " water and condensed steam	=	25·6° C.
Specific heat of copper . . .	=	·095

Answer. 537 nearly.

(9.) An icicle weighing 200 grammes was cooled down to −10° C., and was then immersed in some water at 0° C., the temperature of the surrounding air being also 0° C. The temperature of the icicle rose to 0° C., and on removing it and weighing, it was found to have gained 12·5 grammes. Find from these data the specific heat of ice.

Let x represent the specific heat of ice; then, as the temperature changes from −10° C. to 0° C. the 200 grammes

of ice absorb $200 \times 10 \times x$ heat units. The 12·5 grammes of water in solidifying give out $80 \times 12{\cdot}5$ heat units, and

$$\therefore\ 2000\,x = 80 \times 12{\cdot}5.$$

$$\therefore\ x = \frac{1}{2}.$$

(10.) What number of heat units are required in order to convert 7 lbs. of ice at $-12°$ C. into water at 25° C.?

Answer. 777 heat units.

(11.) How many units of heat are required to convert 3 lbs. of ice at $-6°$ C. into water at 40° C.?

Answer. 369 units.

(12.) What quantity of heat is required in order to convert 20 lbs. of ice at $-12°$ C. into steam at 100° C.?

Answer. 14,460 units.

(13.) What quantity of heat is required to raise a pound of ice at $-10°$ C. to steam at 100° C.?

Answer. 722 pound units.

(14.) A pound of crushed ice which had been cooled down to $-6°$ C. was immersed in a quantity of water at 6° C., and when the ice was all melted the temperature of the mixture was 4·5° C. What was the original quantity of water?

Let x be the number of pounds of water, then
the heat lost by the water $= x \times 1{\cdot}5$ units.
" gained " ice $= 3 + 80 + 4{\cdot}5 = 87{\cdot}5$ units.

$$\therefore\ x = \frac{87{\cdot}5}{1{\cdot}5} = 58\tfrac{1}{3}\text{ lbs.}$$

(15.) A lump of ice at $-8°$ C. was placed in a vessel containing 20 lbs. of water at 15° C., and when the ice had all melted the temperature was 6° C. What was the quantity of ice? *Answer.* 2 lbs.

(16.) How many pounds of steam at 100° C. will be required to melt 50 lbs. of ice at $-6°$ C.?

Answer. 6·515 lbs. nearly.

(17.) If twenty cubic metres of air at 12° C. and 76·2 centimetres pressure, and of which the hygrometric state is ·62, be mixed with 36 cubic metres of air at 16° C. and 77 centimetres pressure, and of which the hygrometric state is ·4, find the temperature of the mixture and its hygrometric state, having given that the final bulk is 56 cubic metres, and that

maximum pressure of water vapour at 16° C. = 1·353 centims.
" " " " 12° C. = 1·046 "

Specific heat of water vapour is twice that of air.

In solving this question we must first find the respective quantities of dry air and of aqueous vapour in each of the given volumes of moist air.

First quantity.—The pressure of the vapour = ·62 × 1·046 = ·649 centimetre.

$$\text{Dry air} = 20 \times 1293 \times \frac{76{\cdot}2 - {\cdot}649}{76} \times \frac{273}{285} = 24624{\cdot}8 \text{ grms.}$$

$$\text{Vapour} = \frac{5}{8} \times 20 \times 1293 \times \frac{{\cdot}649}{76} \times \frac{273}{285} = \quad 132{\cdot}2 \;\;\text{"}$$

Second quantity.—The pressure of the vapour = 1·353 × ·4 = ·541 centimetre.

$$\text{Dry air} = 36 \times 1293 \times \frac{77 - {\cdot}541}{76} \times \frac{273}{289} = 44236{\cdot}5 \text{ grms.}$$

$$\text{Vapour} = \frac{5}{8} \times 36 \times 1293 \times \frac{{\cdot}541}{76} \times \frac{273}{289} = \quad 195{\cdot}6 \;\;\text{"}$$

If $t°$ C. be the temperature of the mixture, and s the specific heat of air,

$$\left.\begin{array}{l}\text{the heat gained by}\\ \quad\text{the cooler air}\;\;.\end{array}\right\} = (24624{\cdot}8 + 2 \times 132{\cdot}2)s(t - 12) \text{ units,}$$

$$\left.\begin{array}{l}\text{the heat lost by the}\\ \quad\text{warmer air}.\;\;.\end{array}\right\} = (44236{\cdot}5 + 2 \times 195{\cdot}6)\,s(16 - t) \text{ units,}$$

and since these quantities of heat must be equal, we have

$$\frac{t - 12}{16 - t} = \frac{44627{\cdot}7}{24889{\cdot}2} = 1{\cdot}793,$$

whence $t = 14{\cdot}56°$ C.

Next, to calculate the hygrometric state of the air, we find from a table of vapour pressures that the maximum pressure of aqueous vapour at 14·56° C. is 1·235 centimetre, and the bulk of the mixture being 56 cubic metres the quantity of water vapour which *would* saturate this at 14·56° C. is

$$\frac{5}{8} \times 56 \times 1293 \times \frac{1 \cdot 235}{76} \times \frac{273}{287 \cdot 56} = 699 \cdot 8 \text{ grammes.}$$

But the quantity of aqueous vapour actually present $= 132 \cdot 2 + 195 \cdot 6 = 327 \cdot 8$ grammes.

Therefore the hygrometric state of the mixture

$$= \frac{327 \cdot 8}{699 \cdot 8} = \cdot 468.$$

(18.) Three cubic metres of moist air, the temperature of which is 15° C., the pressure 76 centimetres, and hygrometric state ·82, are mixed with 2 cubic metres of moist air at 5° C., 76 centimetres pressure, and hygrometric state ·8, and the mixture occupies 5 cubic metres. Having given that the maximum pressures of water vapour at 5°, 10·9°, 15° are respectively ·6, ·98, and 1·27 centimetres of mercury, find the hygrometric state of the mixture. *Answer.* ·83.

CONDUCTION.

(1.) A plate of wrought iron 2 centimetres thick and 10 square decimetres in area was placed so as to form a partition separating water, which was kept at 15° C. on the one side, from melting ice on the other, and it was found that in one hour 59·2 kilogrammes of ice were melted. Deduce from this the conductivity of wrought iron.

The *thermal conductivity* of a substance is measured by the number of heat units which pass in one second across a plate of the substance of unit area and unit thickness when its faces differ in temperature by 1° C.

Let κ = coefficient of conductivity in centimetre-gramme-second units.

A = area of the plate in square centimetres.

d = thickness of the plate in centimetres.

t = temperature of cold side of the plate.

t' = " " hot " "

n = number of seconds during which the flow of heat proceeds.

Q = number of heat units which flow across the area A in n seconds.

Then
$$Q = \kappa \times n \times (t' - t) \times \frac{A}{d}.$$

$$\therefore \kappa = \frac{Q \times d}{n(t' - t)A}.$$

In the present case

$$Q = 59200 \times 80 \text{ units,}$$
$$d = 2$$
$$n = 3600$$
$$t' - t = 15$$
$$A = 1000$$

$$\therefore \kappa = \frac{59200 \times 80 \times 2}{3600 \times 15 \times 1000} = \cdot 175.$$

(2.) One side of a brass plate one centimetre thick and one square decimetre in area is kept in contact with boiling water, and the other side with melting ice, and it is found that in 8 minutes 64·92 kilogrammes of ice were melted. Find the conductivity of brass in c.-g.-s. units.

Answer. 1·082.

(3.) How much water will be evaporated per hour when it is boiled at 100° C. in an iron boiler 1·5 centimetre thick, having the area of its heating surface equal to 460 square centimetres, and its outer surface being kept at 180° C.?

By the formula in Example 1 we have

$$Q = \kappa \times n \times (t' - t) \times \frac{A}{d} = \cdot 175 \times 3600 \times 80 \times \frac{460}{1 \cdot 5} \text{ heat units.}$$

And if x = number of grammes of water evaporated per hour, taking 537 for the latent heat of steam, we have

$$x = \frac{Q}{537} = \frac{\cdot175 \times 3600 \times 80 \times 460}{537 \times 1\cdot5} = \begin{cases} 28782 \text{ grammes, or} \\ 28\cdot782 \text{ kilogrammes.} \end{cases}$$

(4.) The temperature of a large room is kept at 21° C. by a large iron stove, the temperature of the interior of which is kept constant at 200° C. If the thickness of the stove be one centimetre, find how much heat will be given off per minute from each square decimetre of its exterior face.

Answer. 187·95 kilogramme-degrees.

(5.) The brick walls of a cottage are 50 centimetres thick, and the inside of the cottage is kept constantly at 16° C., while the temperature outside is −5° C. How much heat will be lost per hour from each square decimetre of the outer face of the walls? The conductivity of the brick is ·0034. *Answer.* 514·08 water-gramme-degrees.

(6.) Find the quantity of heat which is given off per minute from each square decimetre of the surface of an iron steam boiler 8 millimetres thick, when the temperature of the inner surface of the boiler is kept at 120° C. and that of the outer surface is 119·5° C.

Answer. 656·25 gramme-degrees.

(7.) On a certain day the temperature of the hot-room at a Turkish bath was 52° C., and that of the cold-room 18° C., and the two rooms were separated from each other by a plate of glass 2 centimetres thick. The lower portion of this glass was in the form of a rectangle 4·3 metres high and 2·4 metres broad, while the upper part was in the shape of three-quarters of a circle of 1·2 metre radius. Find the quantity of heat given off per minute by this plate of glass, the conductivity of glass in c.-g.-s. units being ·015.

Answer. 2251·288 kilogramme-degrees.

FORCE OF EXPANSION AND CONTRACTION.

(1.) An iron bar whose sectional area is 2 square inches is allowed to cool from 100° C. to 0° C. What force would it exert if it were prevented from contracting?

Let l = natural length of bar at 0° C.

Then $l(1 + 100a)$ = " " " 100° C.

$$\therefore \frac{\text{Increment in length}}{\text{Original length}\ .\ .} = 100\,a = \cdot 001225.$$

Hence, if the bar be prevented from contracting when its temperature changes from 100° C. to 0° C., it is stretched beyond its natural length at 0° C. by a quantity $l \times \cdot 001225$.

Taking 29,000,000 lbs. as the modulus of elasticity for iron, we shall find that the force required to produce this elongation in a bar of 2 square inches section is

$$F = \frac{29000000}{20 \times 112} \times 2 \times \cdot 001225 \text{ tons.}$$
$$= 31 \text{ tons } 14 \text{ cwt. } 42 \text{ lbs.}$$

And this represents the force with which the bar tends to contract.

(2.) What force would be required to prevent the expansion of an iron bar half of a square inch in section if its temperature were raised from 10° C. to 30° C., the coefficient of expansion of iron being ·000012?

Answer. 1 ton 11 cwt. 8 lbs.

(3.) A cast-iron pillar 12 square inches in section is firmly fixed between two immovable blocks, the temperature being 0° C. What will be the pressure exerted against these blocks if the temperature rises to 30° C. and the modulus of elasticity for cast iron be 17,000,000 lbs.?

Answer. 32 tons 15 cwt. 80 lbs.

(4.) When an iron rod 33 feet 4 inches long and 3 inches in diameter is stretched by a force of 37·5 tons, the elongation is ·171 of an inch. What variation in its temperature

would produce the same effect, the coefficient of linear expansion of the iron being $\cdot 0000122$ for $1°$ C.?

Let $x°$ C. be the required variation in the temperature; then $$\cdot 171 = 400 \times \cdot 0000122 \times x.$$

$$\therefore x = \frac{\cdot 171}{400 \times \cdot 0000122} = 35\cdot 041.$$
$$= 35° \text{ C. nearly.}$$

(5.) An iron bar, 10 feet long and $\frac{1}{8}$th of a square inch in section, is elongated half an inch by a stretching-force of 17,286 lbs. What change of temperature would produce the same effect? *Answer.* 342° C. nearly.

(6.) According to Wertheim's experiments, the mean force which is required to stretch an iron bar of one square centimetre section by one-millionth of its length, when its temperature is between 0° C. and 100° C., is 2,125 grammes. If the coefficient of expansion of iron be $\cdot 000012$, calculate the requisite change of temperature to produce the same effect. *Answer.* $\frac{1}{12}°$ C.

(7.) The upper end of an iron wire, whose sectional area is $\cdot 021$ of a square centimetre, is rigidly attached to a beam, and to the lower end is fastened a scale-pan resting on the ground. What weight will have to be put into the scale-pan so as to keep it on the ground if the temperature falls through 25° C.?

The contraction would be $25 \times \cdot 000012 = \cdot 0003$ of the length of the wire, but a contraction of one-millionth would require an opposing force of $2125 \times \cdot 021 = 44\cdot 625$ grammes. $\therefore$ a contraction of 300 millionths requires an opposing force of $44\cdot 625 \times 300 = 13387\cdot 5$ grammes.

(8.) If the sectional area of the wire had been $\cdot 006$ of a square centimetre, and the fall of temperature from 20° C. to 7° C., what would have been the requisite opposing weight? *Answer.* 1,989 grammes.

(9.) A series of iron rods one square inch in section were carried across an old church, passing through holes in the

walls, and secured on the outside by screw-nuts and washers. The rods were then heated from 10° C. to 120° C., the nuts screwed home, and the rods then allowed to cool. The initial length of each rod was 40 feet, and the holes were 16 feet above the joint in the masonry about which the walls were to turn. The walls being initially 4° out of the vertical, calculate how often the rods would have to be heated so as to render the walls vertical.

The distance of each wall from the central line towards which it had to be pulled was 20 feet, and the arc through which the top of the wall had to be drawn was

$$\frac{\pi}{45} \times 16 \text{ feet.}$$

The expansion of an iron rod 20 feet long for a change of temperature of 110° C. is $\cdot 000012 \times 20 \times 110 = \cdot 0264$ of a foot, and if x be the number of times the rods require to be heated, we shall have

$$x \times \cdot 0264 = \frac{\pi \times 16}{45}.$$

$$\therefore x = \frac{\pi \times 16}{45 \times \cdot 0264} = 42 \cdot 311.$$

$$= 43 \text{ times nearly.}$$

(10.) If the walls had been sixty feet apart, 2° out of the vertical, and the holes 20 feet above the line of rotation, what number of heatings would have been required?

Answer. 17·63, or 18 times nearly.

(11.) According to Grassi's investigations, a pressure of 343,644 dynes per square centimetre is required to compress mercury by one-millionth of its bulk. What change of temperature would produce the same result?

When the temperature rises from 0° C. to 100° C. the expansion of mercury is ·018153 of its bulk at 0° C., and

$\therefore$ for $1°$ C. it expands by $\cdot 00018153$, or $181\cdot 53$ millionths. Hence, if $x°$ C. be the required change of temperature,

$$x \times 181\cdot 53 = 1.$$

$$\therefore x = \frac{1}{181\cdot 53} \text{ of a degree Centigrade.}$$

(12.) If mercury were heated from $0°$ C. to $100°$ C., what pressure would be required to prevent its expanding?

The change of volume $= 100 \times \cdot 00018153$ or 18153 millionths.

$\therefore$ pressure required

$$= 343644 \times 18153 \text{ dynes per square centimetre.}$$

$$= \frac{343644 \times 18153}{9\cdot 81 \times 10^5} \text{ kilogrammes per square centimetre.}$$

$$= 6359 \text{ kilogrammes per square centimetre.}$$

$$= \frac{6359}{1\cdot 01605 \times 10^3} \text{ tons per square centimetre.}$$

$$= 6\cdot 25 \text{ tons per square centimetre nearly.}$$

(13.) Water experiences a compression of 50 millionths of its bulk for a pressure equal to that of the atmosphere, which is about $1\cdot 014 \times 10^6$ dynes per square centimetre. Water expands by $8\cdot 2$ millionths of its bulk at $4°$ C. when its temperature rises from $4°$ C. to $5°$ C. What pressure per square centimetre would be required to prevent this expansion? *Answer.* 166,296 dynes.

(14.) If the expansion of mercury between $0°$ C. and $10°$ C. is $\cdot 0017905$ of its volume at $0°$ C., calculate the pressure per square centimetre which would be required to prevent its expansion if its temperature were raised by this amount? *Answer.* $627\cdot 21$ kilogrammes.

THERMODYNAMICS.

(1.) Find the distance through which a mass of one ton can be raised against the force of gravity by the expenditure of a quantity of heat equal to that which would raise the temperature of one pound of water from 0° C. to 15° C.

Joule's experiments have proved that the quantity of work which has to be done to raise a mass of 1,390 lbs. against gravity through a vertical height of one foot would be sufficient, if it were entirely converted into heat, to raise the temperature of one pound of water from 0° C. to 1° C. Hence the 'dynamical equivalent,' or, as it is often called, 'Joule's equivalent,' of this quantity of heat is 1,390 foot-pounds.

If Q represent the quantity of heat in water-pound-degrees Centigrade,
W " " work which can be done by this heat in foot-pounds,
J " Joule's equivalent,

then, by the first law of Thermodynamics,

$$W = Q \times J = 15 \times 1390 = 20850 \text{ foot-pounds, or } 9{\cdot}3 \text{ foot-tons nearly.}$$

(2.) A mass of one ton is lifted by a steam-engine to a height of 200 feet. What is the quantity of heat expended in doing this?

$$Q = \frac{W}{J} = \frac{20 \times 112 \times 200}{1390} = 322{\cdot}3 \text{ pound-units nearly.}$$

(3.) How much heat is expended in lifting a mass of 3 hundredweight through a vertical distance of 250 feet?

Answer. 60·432 pound-units.

(4.) A man weighing 12 stone ascends a tower 90 feet high. How much heat has his body to supply in doing this?

Answer. 10·87 pound-units.

(5.) Find the heat-equivalent of one horse-power in water-pound-degrees Centigrade.

One theoretical horse-power = 33,000 foot-pounds per minute.

$\therefore$ heat-equivalent $= \frac{33000}{1390} = 23{\cdot}741$ pound-degrees per minute.

(6.) When water is the standard substance, the dynamical equivalent of a pound-degree Centigrade of heat is 1,390 foot-pounds; what would be the value of the 'dynamical equivalent' of a pound-degree if the standard substance were mercury, the specific heat of which is ·032?

Answer. $1390 \times {\cdot}032 = 44{\cdot}48$ foot-pounds.

(7.) The mass of a train, including the engine, is 100 tons, and the resistance is 8 pounds per ton. What will be the least quantity of heat (in water-pound-degrees Centigrade) which will have to be expended by the steam in a run of 100 miles on a level road?

$$\text{Work done} = 800 \times 100 \times 1760 \times 3 \text{ foot-pounds.}$$

$$\therefore \text{ heat expended} = \frac{800 \times 100 \times 1760 \times 3}{1390} = 303885 \text{ units.}$$

(8.) A lump of lead weighing 16 pounds falls from a height of 200 feet on to a hard, non-conducting surface. If all the heat generated by the collision be absorbed by the lead, find its rise of temperature, the specific heat of lead being ·03.

When a falling body has its motion suddenly arrested, the quantity of heat generated by the stoppage of its motion is such that its dynamical equivalent would have just sufficed to lift the body back to its original position. Now, the dynamical equivalent of one water-pound-degree Centigrade of heat will lift 1,390 pounds one foot high, or one pound 1,390 feet.

$\therefore$ the heat-equivalent of the stoppage of the motion of one pound after a fall of 1,390 feet = one water-pound-degree;

$\therefore$ heat-equivalent of 16 pounds after falling 200 feet

$$= \frac{16 \times 200}{1390} \text{ water-pound-degrees Centigrade.}$$

$$\therefore \text{ rise of temperature } = \frac{16 \times 200}{1390 \times 16 \times \cdot 03} = 4 \cdot 8^\circ \text{ C. nearly.}$$

(9.) An iron cannon-ball of 68 pounds is dropped from a tower 350 feet high on to a hard, non-conducting surface at its base. If the specific heat of iron be ·1138, and all the heat generated by the collision be absorbed by the ball, by how much will its temperature be raised?

Answer. 2·2° C. nearly.

(10.) A platinum ball falls from a height of 80 metres on to a rigid, inelastic surface. If all the heat be absorbed by the ball, and the specific heat of platinum be ·032, find the rise of temperature. *Answer.* 5·9° C. nearly.

(11.) A block of stone weighing a ton falls upon a glacier from a height of 800 feet. If one quarter of the heat generated by the stoppage of its motion enters the ice, how much ice will be melted?

Let x be the number of pounds of ice melted; then, since the heat generated by the fall

$$= \frac{20 \times 112 \times 800}{1390} = 1289 \cdot 2 \text{ water-pound-degrees C.}$$

$$\therefore x \times 80 = \frac{1289 \cdot 2}{4} \quad \therefore x = 4 \cdot 03 \text{ pounds nearly.}$$

(12.) From what height would a block of ice at 0° C. have to fall so that the heat generated by its collision with the earth should be just sufficient to melt it? From what height would it have to fall that the heat generated might be sufficient to convert it into steam?

Answers. 111,200 feet and 996,630 feet.

(13.) By what fraction of a degree Centigrade is the temperature of the water of Niagara raised by its fall, the height being 164 feet? *Answer.* 0·12° C. nearly.

(14.) What must be the height of a waterfall that the temperature of the water may be raised 1° F.?

Answer. 772 feet nearly.

(15.) What is the numerical value of Joule's equivalent when the kilogrammetre is taken as the unit of work?

Since one foot = ·3048 metre,

$$J = 1390 \times \cdot 3048 = 423 \cdot 67 = 424 \text{ nearly.}$$

N.B. The unit quantity of heat is in this case the quantity required to raise one kilogramme of water from 0° C. to 1° C.

(16.) Some mercury drops from one vessel into another vessel 15 feet below it. By what fraction of a degree Centigrade will its temperature be raised, the specific heat of mercury being ·033? *Answer.* $\frac{1}{3}$° C. nearly.

(17.) One kilogramme of water at 100° C. and at a pressure of 1,033·3 grammes per square centimetre is converted into 1,695 litres of steam at 100° C. Taking the latent heat of steam at 537 and J = 424, calculate how much of the heat is spent in internal and how much on external work.

At 100° C. one kilogramme of water occupies 1·043 litres.

" " " steam " 1695· "

∴ increase of bulk = 1693·957 litres = 1·693957 cubic metres.

Also a pressure of 1033·3 grammes per square centimetre corresponds to 10,333 kilogrammes per square metre, hence the external work done = 1·693957 × 10333 kilogrammetres,

and the heat-equivalent $= \dfrac{1 \cdot 693957 \times 10333}{424} = 41 \cdot 282$ units.

The heat expended on internal work

$$= 537 - 41 \cdot 282 = 495 \cdot 718 \text{ units.}$$

(18.) How much external work is done when one gramme of water at 100° C. is converted into steam at 100° C.? Given that

At 100° C. one gramme of water occupies 1·0432 c.c.

" " " steam " 1773·4 c.c.

Answer. 18·3142 kilogrammetres.

(19.) How much work, in foot-pounds, is done when a pound of water is converted into a pound of steam? Given that

Pressure of steam = 14 pounds to the square inch.
One cubic foot of water weighs 62·5 pounds.
One cubic inch of water produces one cubic foot of steam. *Answer.* 55,706 foot-pounds.

(20.) A cube of iron 1 decimetre in the edge is heated up from the freezing to the boiling point of water. Taking the coefficient of linear expansion of iron for 1° C. at ·0000122 and the pressure of the air at 1,033 grammes per square centimetre, find the external work done by the cube in expanding. *Answer.* 3,780 gramme-centimetres.

(21.) Express in metre-kilogramme units the quantity of work which would have to be expended in order to compress 1 gramme of steam at 100° C. and 76 centimetres pressure into water at 100° C. *Answer.* $\frac{537 \times 424}{1000} = 227{\cdot}688.$

(22.) Having given

(1) Density of mercury at 0° C. . = 13·596
(2) Mass of 1 litre of air at 0° C. and 76 centimetres pressure = 1·2932 grms.
(3) Specific heat of air at constant pressure . . . . = ·237
(4) Specific heat of air at constant volume . . . . = ·167

find the value of the dynamical equivalent of heat.

At 0° C. and 76 centimetres pressure 1 gramme of air occupies . . $\frac{1000}{1{\cdot}2932}$

$= 773{\cdot}28$ c.c. nearly.

At 1° C. and 76 centimetres pressure 1 gramme of air occupies . . $773{\cdot}28\left(1 + \frac{1}{273}\right)$

$= 776{\cdot}11$ c.c.

∴ increment of bulk = 2·83 c.c.

$$\left.\begin{array}{l}\text{The pressure of the air in grammes}\\ \quad\text{per square centimetre}\quad .\quad .\end{array}\right\} = 76 \times 13{\cdot}596$$

$$= 1033{\cdot}3 \text{ grammes.}$$

$\therefore$ the work done by the air in expanding

$$= 1033{\cdot}3 \times 2{\cdot}83 = 2924{\cdot}2 \text{ gramme-centimetres.}$$

But the heat absorbed in doing this

$$= {\cdot}237 - {\cdot}167 = {\cdot}07 \text{ gramme-degree.}$$

Hence the dynamical equivalent of 1 water-gramme-degree Centigrade of heat, as deduced from this experiment, is

$$\frac{2924{\cdot}2}{{\cdot}07} = 41774 \text{ gramme-centimetres.}$$

And $\therefore$ the dynamical equivalent of 1 kilogramme-degree Centigrade is

41774 kilogramme-centimetres,
or 417·74 kilogramme-metres.

(23.) A cubic metre of air at 0° C. and 76 centimetres pressure is heated up to 100° C. at constant pressure. Find the quantity of heat expended on external work, having given that

Density of mercury at 0° C. . . = 13·596
Specific heat of air at constant pressure = ·237
Dynamical equivalent of one water-gramme-degree C. . . . = 42813 gr. cms.

Since 1 cubic metre of air at 0° C. and 76 centimetres pressure contains 1,293 grammes, the quantity of heat required to raise its temperature to 100° C. at constant pressure is

$$1293 \times 100 \times {\cdot}237 = 30644 \text{ gramme-degrees.}$$

The increment of volume is $\frac{100}{273}$ cubic metre, or $\frac{10^8}{273}$ c.c.

Hence the work done in pressing back the surrounding air is

$$76 \times 13{\cdot}596 \times \frac{10^8}{273} \text{ gramme-centimetres,}$$

and the heat-equivalent of this work is

$$\frac{76 \times 13{\cdot}596 \times 10^8}{273 \times 42813} = 8840{\cdot}7 \text{ gramme-degrees.}$$

Hence the fraction of the total heat which is expended on external work is

$$\frac{8840{\cdot}7}{30644} = {\cdot}29 \text{ nearly.}$$

(24.) One gramme of air at 0° C. and 76 centimetres pressure is heated to 1° C. at constant pressure. With the data of Example 23, calculate the quantity of heat which is expended on external work. *Answer.* ·068 of a w.-g.-d. C.°

(25.) What is the value of the real specific heat of air at constant pressure?

The real specific heat of air at constant pressure, being the quantity of heat actually absorbed in raising the *temperature* of 1 gramme of air from 0° C. to 1° C., is

$${\cdot}237 - {\cdot}068 = {\cdot}169 \text{ w.-g.-d. C.}^\circ$$

(26.) What is the ratio of the apparent to the real specific heat of air at constant pressure? *Answer.* 1·402.

(27.) The density of hydrogen gas at 0° C. and 76 centimetres pressure is ·0000896 gramme. Assuming that the acceleration of gravity is 981 centimetres per second, what is the average velocity of agitation of the molecules of hydrogen gas?

If v be the velocity of mean square of the molecules of a gas, p the pressure, and d the density, all in centimetre-gramme-second units, then by Maxwell's 'Heat,' p. 294, 3rd edition,

$$v^2 = \frac{3p}{d}.$$

In the present case $d = {\cdot}0000896$; $p = 76 \times 13{\cdot}596 \times 981$ absolute units of force.

$$\therefore v = \sqrt{\frac{3 \times 76 \times 13{\cdot}596 \times 981}{{\cdot}0000896}} = 184227 \text{ centimetres}$$

per second.

(28.) A litre of air at 0° C. and 1033·3 grammes pressure per square centimetre contains 1·293 grammes. Find the average velocity of the air-particles.

Answer. 484·96 metres per second.

(29.) The density of oxygen gas at 0° C. and 76 centimetres pressure is ·00143 gramme. Find the average velocity of the molecules of oxygen gas at 50° C. and the same pressure.

The density of oxygen gas at 50° C.

$$= \frac{\cdot 00143 \times 273}{323} = \cdot 001208 \text{ nearly.}$$

$$\therefore v = \sqrt{\frac{3 \times 76 \times 13\cdot 596 \times 981}{\cdot 001208}} = 50160 \text{ centimetres}$$

per second.

(30.) What is the average velocity of the molecules of oxygen gas at 76 centimetres pressure and at the temperature of melting ice? *Answer.* 461·15 metres per second.

(31.) What is the average velocity of the oxygen molecules at the same pressure but at 80° C.?

Answer. 524·23 metres per second.

(32.) Assuming that oxygen were still a perfect gas at — 200° C., what would be the average velocity of the molecules at the same pressure as before?

Answer. 238·47 metres per second.

(33.) The work done by one theoretical 'horse' per minute is 33,000 foot-pounds. Express this in 'ergs' per second.

The centimetre-gramme-second (C.-G.-S.) unit of force, which is called a *dyne*, is that force which, acting on a gramme of matter for 1 second, generates in it a velocity of 1 centimetre per second. The force of gravity in London may be taken as 981 such units.

The C.-G.-S. unit of work is called an *erg*. The work done in raising a gramme through 1 centimetre against gravity is 981 ergs.

Also 1 foot = 30·4797 centimetres; 1 pound = 453·59 grammes.

$$\therefore \text{ one horse-power} = \frac{33000}{60} \text{ foot-pounds per second.}$$

$$= \frac{33000 \times 30{\cdot}4797 \times 453{\cdot}59 \times 981}{60} \text{ ergs per second.}$$

$$= 7{\cdot}4594 \times 10^9 = 7{\cdot}46 \times 10^9 \text{ ergs per second nearly.}$$

(34.) From the data of Example 22 we found that the dynamical equivalent of 1 water-gramme-degree Centigrade of heat is 41,774 gramme-centimetres. Express this in *absolute* units.

Answer. One w.-g.-d. C.° $= 41774 \times 981$
$= 4{\cdot}098 \times 10^7$ ergs.

(35.) In practice the dynamical equivalent of 1 w.-g.-d. C.° of heat is usually taken to be $4{\cdot}2 \times 10^7$ ergs. Find the dynamical equivalent of 1 water-pound-degree Centigrade of heat. *Answer.* $1{\cdot}91 \times 10^{10}$ ergs nearly.

(36.) A bullet weighing 250 grammes strikes a target with a velocity of 300 metres per second. Express in water-kilogramme-degrees Centigrade the quantity of heat which will be generated by the collision.

The kinetic energy of the bullet at the moment of striking the target, being equal to one-half the product of its momentum and its velocity, is

$$\frac{mv^2}{2} = \frac{250 \times (30000)^2}{2} \text{ ergs.}$$

And by the data of Example 35 the dynamical equivalent of 1 kilogramme-degree Centigrade of heat is $4{\cdot}2 \times 10^{10}$ ergs.

$\therefore$ the quantity of heat developed by the collision is

$$\frac{250 \times (30000)^2}{2 \times 4{\cdot}2 \times 10^{10}} = 2{\cdot}679 \text{ units nearly.}$$

(37.) A 1,700-pound shot from an 80-ton gun strikes a target with a velocity of 1,496 feet per second. Express in

water-pound-degrees Centigrade the quantity of heat generated by the collision.

The acceleration of gravity being 32·2 foot-second units, the dynamical equivalent of 1 water-pound-degree Centigrade in *absolute* measure is 1390 × 32·2 units, and

∴ the heat generated by the collision is

$$\frac{1700 \times (1496)^2}{2 \times 32{\cdot}2 \times 1390} = 42502 \text{ w.-p.-d. C.}^\circ$$

(38.) A 700-pound shot strikes an armour plate with a velocity of 1,600 feet per second. What quantity of heat is developed by the blow? *Answer.* 20,019 w.-p.-d. C.°

(39.) Herschell calculated that the average velocity of a shooting star is 35 miles per second, and its average mass 2 ounces. When its motion is suddenly arrested what quantity of heat is developed?

The kinetic energy of the moving mass expressed in foot-pound-second units is

$\frac{1}{8} \times \frac{(35 \times 1760 \times 3)^2}{2}$ units, and the heat-equivalent of this is

$$\frac{(35 \times 1760 \times 3)^2}{8 \times 2 \times 32{\cdot}2 \times 1390} = 47688 \text{ w.-p.-d. C.}^\circ$$

(40.) The specific heat of iron being 0·11, find the velocity with which a mass of iron must strike a hard non-conducting surface, so as to have its temperature raised 1° C.

Let m = mass of iron in grammes,

v = velocity in centimetres per second,

then its kinetic energy $= \frac{mv^2}{2}$ ergs,

and the heat equivalent of this $= \frac{mv^2}{2 \times 4{\cdot}2 \times 10^7}$ w.-g.-d. C.°

∴ the rise of temperature $= \frac{v^2}{2 \times 4{\cdot}2 \times 10^7 \times {\cdot}11} = 1$ by the question.

From this equation we get

$v = 3039{\cdot}74$ centimetres per second.

(41.) The melting-point of lead being 326° C., the specific

heat of solid lead ·0314, and the latent heat of fusion 5·4, find the velocity with which a leaden bullet must strike a target so that if all the heat generated by the collision were absorbed by the bullet it might be completely melted, its initial temperature being 10° C.

Answer. 358·76 metres per second.

(42.) The mass of a railway truck and its contents is 10 tons and the resistance is 7·5 pounds a ton. It is drawn from rest by a horse, and after going 250 feet is observed to be moving at the rate of 4 feet per second. How much heat has been expended by the horse in doing this work?

The kinetic energy of the moving truck

$$= \frac{10 \times 20 \times 112 \times 16}{2 \times 32{\cdot}2} = 5565{\cdot}2 \text{ foot-pounds.}$$

The work done in moving 10 tons 250 feet

$$= 75 \times 250 = 18750 \text{ foot-pounds.}$$

$\therefore$ total work done by the horse

$$= 5565{\cdot}2 + 18750 = 24315{\cdot}2 \text{ foot-pounds;}$$

and the heat-equivalent of this is

$$\frac{24315{\cdot}2}{1390} = 17{\cdot}49 \text{ w.-p.-d. C.}^\circ$$

(43.) How much heat is developed in stopping a railway train of 60 tons which is running on the level at the rate of 45 miles an hour? *Answer.* 6540·1 w.-p.-d. C.°

(44.) The mass of a railway train is 100 tons and the friction is 7 pounds per ton. How many units of heat must be expended in drawing it for 4 miles up an incline of 1 in 200?

Since 1 mile = 5280 feet and 1 ton = 2240 pounds,

The work done against gravity

$$= 100 \times 2240 \times 105{\cdot}6 = 23654400 \text{ foot-pounds.}$$

The work done against friction

$$= 100 \times 7 \times 4 \times 5280 = 14784000 \text{ foot-pounds.}$$

$\therefore$ total work = 38438400 foot-pounds.

And the heat equivalent

$$= \frac{38438400}{1390} = 27654 \text{ w.-p.-d. C.}^\circ \text{ nearly.}$$

(45.) The mass of the earth being $6\cdot069 \times 10^{21}$ tons, and its average rate of translation round the sun 33,290 yards per second, find the number of tons of coal which would have to be burnt to produce a quantity of heat equal to that which would be developed by the sudden stoppage of the earth's motion of translation.

The heat-equivalent of the kinetic energy is, according to the method of Example 39,

$$\frac{10^{21} \times (6\cdot069 \times 2240) \times (3 \times 33290)^2}{2 \times 32\cdot2 \times 1390} \text{ w.-p.-d. C.}^\circ$$

and *assuming* that a pound of coal in burning gives out 8,000 pound-units of heat, the required quantity of coal is

$$\frac{10^{21} \times (6\cdot069 \times 2240) \times (3 \times 33290)^2}{2 \times 32\cdot2 \times 1390 \times (8000 \times 2240)} = 84\cdot527 \times 10^{21} \text{ tons.}$$

That is to say, the quantity of heat would be equal to that produced by the sudden and complete combustion of a mass of coal about 14 times that of the earth.

(46.) Find the quantity of heat which would be developed by the sudden stoppage of the rotation of the earth.

The moment of inertia of a sphere of mass m and radius r which is rotating about a diameter is $\frac{2}{5} mr^2$, and if ω be the angular velocity of a body rotating about an axis passing through its centre of gravity, then (*vide* Routh's 'Rigid Dynamics,' § 194, edition of 1860)

$$\Sigma mv^2 = w^2 \text{ MK}^2.$$

Hence, if r be expressed in feet and m in pounds, the kinetic energy of the earth in virtue of its rotation upon its axis is

$$\frac{2}{5} \times \frac{mr^2\omega^2}{2 \times 32\cdot2}. \quad \text{But } \omega = \frac{2\pi}{24 \times 60 \times 60}.$$

$$\therefore \text{ kinetic energy} = \frac{2}{5} \times \frac{mr^2}{2 \times 32\cdot2} \times \frac{4\pi^2}{(24 \times 60 \times 60)^2},$$

and then, proceeding as in Example 45, the number of tons of coal required is

$$\frac{10^{21} \times (6\cdot069 \times 2240) \times (4000 \times 1760 \times 3)^2 \times \pi^2}{5 \times 32\cdot2 \times (12 \times 3600)^2 \times 1390 \times 1792 \times 10^4}$$
$$= 7\cdot9966 \times 10^{18} \text{ tons.}$$

HEAT-ENGINES.

(1.) In a certain heat-engine the temperature of the source is 130° C. and that of the refrigerator is 22° C. Supposing that the engine is a 'perfect engine,' what is its efficiency?

If W be the work done by an engine when a quantity of heat H is abstracted from the source, both being estimated in dynamical measure, then the ratio $\frac{W}{H}$ is termed the 'efficiency' of the engine. If T and t be the absolute temperatures of the source and refrigerator, then (Maxwell's 'Heat,' p. 160, 3rd edition),

$$\frac{W}{H} = \frac{T - t}{T}$$
$$= \frac{130 - 22}{130 + 273} = \frac{108}{403} = \cdot268 \text{ nearly.}$$

(2.) Find the maximum efficiency of an engine working between the temperatures 148° C. and 48° C.

Answer. ·238.

(3.) An engine which is to be considered as a 'perfect engine' is supplied with heat from a source at 200° C., and gives out heat to a condenser at 80° C. If the engine be supposed to work at the rate of 8 horse-power, calculate the quantity of heat which is taken from the source, and the quantity of heat which is given out to the condenser per hour.

The efficiency of this engine is $\frac{200-80}{200+273} = \cdot 2537$.

The work done per hour

$$= 8 \times 60 \times 33000 = 15840000 \text{ foot-pounds.}$$

The heat-equivalent of this

$$= \frac{15840000}{1390} = 11395 \cdot 7 \text{ pound-units.}$$

But if H be the quantity of heat taken from the source, $H \times \cdot 2537$ is converted into work, and therefore $H(1 - \cdot 2537)$ or $H \times \cdot 7463$ is given out to the condenser.

And $H \times \cdot 2537 = 11395 \cdot 7$.

$$\therefore H = \frac{11395 \cdot 7}{\cdot 2537} = 44918 \text{ pound-units,}$$

and the quantity of heat given out to the condenser is

$$H \times \cdot 7463 = 44918 \times \cdot 7463 = 33522 \text{ pound-units.}$$

(4.) An air-engine takes in heat at 350° F. and gives out heat at 40° F. Assuming it to be a perfect reversible engine, what is its 'efficiency'? *Answer.* ·383 nearly.

(5.) A certain steam-engine which is assumed to be a 'perfect' engine works between the temperatures 170° C. and 80° C. How much work can be done by the withdrawal of 100 pound-degrees of heat from the boiler?

$$\text{The efficiency} = \frac{170-80}{170+273} = \cdot 2032,$$

and the dynamical equivalent of the heat utilised is

$$\cdot 2032 \times 100 \times 1390 = 28244 \cdot 8 \text{ foot-pounds.}$$

(6.) Another engine works between the temperatures 120° C. and 30° C. What is the maximum output of work for 100 thermal units supplied by the boiler?

Answer. 31,832 foot-pounds.

(7.) What proportion of the heat received by the water in the boiler of an engine would be converted into work if

the temperature of the steam were 250° C. and that of the condenser 30° C., supposing the engine to be 'reversible'?

Answer. ·421 nearly.

(8.) A quantity of dry air which occupies 20 litres at 15° C. and 76 centimetres pressure is suddenly compressed so that its pressure rises to 152 centimetres. Assuming that there is no opportunity for the heat to escape, what will be the change in its temperature?

If γ be the ratio of the specific heat of air at constant pressure to its specific heat at constant volume, and p_2, p_1, t_2 and t_1 be the final and initial pressures and absolute temperatures of a quantity of air of which the volume changes without any loss or gain of heat from without, then it is proved in text-books on 'Thermodynamics' that

$$\frac{t_2}{t_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

But $\quad \gamma = 1·408 \therefore \frac{\gamma - 1}{\gamma} = \frac{·408}{1·408} = ·29,$

and $\quad t_1 = 15 + 273 = 288°\text{ C}.$

$$\therefore t_2 = 288 \times (2)^{·29} = 352·1.$$

$$\therefore \text{temperature required} = 352·1 - 273 = 79·1°\text{ C}.$$

(9.) A quantity of gas at 15° C. and 20 atmospheres pressure has its pressure *suddenly* reduced to 1 atmosphere. What will be the fall in its temperature?

Answer. $-167·2°$ C.

(10.) A quantity of dry air is at 0° C. and 76 centimetres pressure. What pressure must be suddenly applied so as to raise its temperature by 1° C.?

If x be the required pressure expressed in centimetres of mercury, then we have

$$\left(\frac{x}{76}\right)^{·29} = \frac{t_2}{t_1} = \frac{274}{273}$$

$$\therefore x = 76\left(\frac{274}{273}\right)^{\frac{1}{·29}} = 76·964.$$

$$\therefore \text{increment of pressure} = ·964 \text{ centimetre of mercury.}$$

(11.) What increase of pressure in grammes per square centimetre does this correspond to?

Answer. $\cdot 964 \times 13 \cdot 596 = 13 \cdot 106$.

(12.) What rise of temperature will be produced in a quantity of gas by suddenly compressing it by $\frac{1}{273}$rd of its bulk at 0° C.?

When a quantity of gas is suddenly compressed without addition or subtraction of heat, the bulk and the temperature are connected by the relation

$$\frac{t_2}{t_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}.$$

But $\frac{v_1}{v_2} = \frac{273}{272}$; $\gamma - 1 = \cdot 408$ and $t_1 = 273°$ C.

$$\therefore t_2 = 273 \times \left(\frac{273}{272}\right)^{\cdot 408} = 273 \cdot 41° \text{ C.}$$

$$\therefore \text{ rise of temperature} = \cdot 41° \text{ C.}$$

(13.) The initial pressure of steam in a cylinder where the stroke is 5 feet is 40 lbs., and expansion commences when 2 feet of the stroke have been performed. Find the pressure at the end of the stroke, and the percentage of gain in the work done by a given quantity of steam in consequence of expansive working.

Let a = length of stroke in feet,

b = distance moved by piston when the steam-supply is cut off,

P = initial pressure of the steam in the cylinder,

x = number of feet described by piston at any part of stroke,

p = corresponding pressure of the steam.

Then *assuming* that the expansion is according to Boyle's law,[1]

$$\frac{p}{P} = \frac{b}{x} \therefore p = \frac{b \times P}{x},$$

[1] Boyle's law requires that the temperature should be constant, and therefore the cylinder must be surrounded by a hot steam-jacket to maintain the temperature of the steam, which loses heat by the work done in its expansion.

and in the present case

$$\left.\begin{array}{l} x = 5 \\ b = 2 \\ \text{P} = 40 \end{array}\right\} \therefore p = \frac{2 \times 40}{5} = 16 \text{ lbs.}$$

In order to find the work done by the steam in expanding, let dx be a very small element of the length of the stroke, throughout which we may consider the pressure as uniform and equal to p; then the work done during this portion of the stroke is

$$d\text{W} = pdx = b \times \text{P} \times \frac{dx}{x},$$

and the whole work done during the expansion of the steam is

$$\text{W} = b \times \text{P} \times \int_b^a \frac{dx}{x} = b \times \text{P} \log._e \frac{a}{b}$$

$$= b \times \text{P} \times 2{\cdot}30258 \times \log._{10} \frac{a}{b},$$

$$= 2 \times 40 \times 2{\cdot}30258 \times \log.\ 2{\cdot}5 = 73{\cdot}3 \text{ foot-pounds,}$$

and the work done before expansion commences is

$$40 \times 2 = 80 \text{ foot-pounds.}$$

$$\therefore \text{the gain per cent. is } \frac{73{\cdot}3 \times 100}{80} = 91\tfrac{5}{8} \text{ per cent.}$$

(14.) If the area of the piston in the last Example is 2,000 square inches, and the number of strokes per minute 31, what is the horse-power?

Total work done per square inch of piston during one stroke is $80 + 73{\cdot}3 = 153{\cdot}3$ foot-pounds.

$\therefore$ total work done on the piston in one minute is

$$153{\cdot}3 \times 2000 \times 31 \text{ foot-pounds.}$$

$$\therefore \text{horse-power} = \frac{153{\cdot}3 \times 2000 \times 31}{33000} = 288{\cdot}02 \text{ horse-power.}$$

(15.) Two steam-engines develope the same power, and have each a 5-feet stroke. The cylinder of one engine is 3 feet 6 inches in diameter, and is worked with a steam

pressure of 45 lbs. per square inch right through the stroke. The other engine is worked with the same initial pressure of steam, but the cut-off is at half-stroke. What must be the diameter of its cylinder?

Work done by first engine per stroke $= 45 \times 5 \times \pi \times (21)^2$ $= 311725$ foot-pounds, and if x be the number of inches in the diameter of the second cylinder, by the method of Example 13 we have for the work done during one stroke

$$\pi \times \frac{x^2}{4} \times 2{\cdot}5 \times 45 \{1 + \log_e 2\}$$

$$= \pi \times \frac{x^2}{4} \times 2{\cdot}5 \times 45 \times 1{\cdot}693147,$$

$$= 311725 \text{ by the question.}$$

$$\therefore x^2 = \frac{4 \times 311725}{\pi \times 2{\cdot}5 \times 45 \times 1{\cdot}693147},$$

whence $x = 45{\cdot}65$ inches nearly, or 3 feet 9·65 inches.

(16.) In a compound engine the average pressure in the large cylinder is 23 lbs., and the diameter is 90 inches, the length of stroke in both cylinders being 5 feet. The steam enters the smaller cylinder at 84 lbs. pressure, and is cut off at $\frac{1}{5}$th of the stroke. What must be the diameter of the smaller cylinder so as to develope the same horse-power as the large one? *Answer.* 65 inches nearly.

(17.) The length of the stroke of a steam-engine is 4 feet 6 inches, the boiler pressure is 10 lbs. above that of the atmosphere, which is 15 lbs., and the steam is cut off after the piston has traversed 2 feet 6 inches. Find the pressure of the steam in the cylinder when it opens to the exhaust, which is 2 inches before the piston arrives at the end of a stroke.

Assuming the exhaust to be perfect, the total pressure of the steam before the supply is cut off is $10 + 15 = 25$ lbs., and the steam pressure is continued for 4 feet 4 inches.

With these data, by the method of Example 13, we find that the required pressure $= 14{\cdot}42$ lbs.

(18.) The length of the stroke of a steam-engine is 5 feet

6 inches, and the boiler pressure is 27 lbs. to the inch. The supply of steam is cut off after the piston has traversed 2 feet. Find the pressure of the steam when it opens to the exhaust, which is 2 inches before the piston arrives at the end of a stroke. *Answer.* $10\frac{1}{8}$ lbs.

(19.) The length of the stroke of an engine is 6 feet, and the pressure of the steam on entering the cylinder is 36 lbs. to the inch. At what point of the stroke should the steam be cut off so that the pressure at the end of the stroke may be 6 lbs. to the square inch?

Let y = number of feet travelled by the piston when steam is cut off.

Then, by the formula in Example 13, we have

$$y = \frac{p \times x}{\text{P}} = \frac{6 \times 6}{36} = 1 \text{ foot.}$$

(20.) Steam enters a cylinder at 72 lbs. pressure, and the length of stroke is 7 feet. If the pressure at the end of the stroke is to be 18 lbs., at what point of the stroke must the steam be cut off? *Answer.* 21 inches.

(21.) The length of stroke is 4 feet, and the pressure at the end of the stroke is 8 lbs., the steam being cut off when the piston has travelled one foot. At what pressure was the steam admitted to the cylinder?

Answer. 32 lbs. to the inch.

(22.) Find the initial pressure of the steam in a cylinder when the final pressure is 12 lbs., the length of stroke 6 feet, and cut-off taking place when the piston has travelled 18 inches. *Answer.* 48 lbs.

(23.) The piston of an engine moves through 18 inches under a pressure of 36 lbs. to the square inch, and the steam is then cut off from the boiler and allowed to expand. What will be the pressure when the piston has moved through another foot?

By the method of Example 13 we have

$$p = \frac{b}{x} \times \text{P} = \frac{18 \times 36}{30} = 21\tfrac{3}{5} \text{ lbs. to the square inch.}$$

(24.) Find the pressure of the steam in a cylinder when the piston has travelled 4 feet, the whole length of stroke being 5 feet, the initial pressure 80 lbs., and cut-off taking place when the piston has completed $\frac{1}{5}$th of the stroke.

Answer. 20 lbs.

(25.) Steam enters a cylinder at 72 lbs. pressure, and is cut off from the boiler when the piston has travelled 16 inches. What will be the pressure of the steam when the piston has moved through another 20 inches?

Answer. 32 lbs.

(26.) The diameter of each cylinder of a locomotive engine is 18 inches, and the length of stroke is 2 feet, the steam being cut off at $\frac{1}{4}$ of the stroke. The steam when admitted to the cylinder is at the pressure of 70 lbs. to the inch, and the crank makes 58 revolutions a minute. Find the horse-power.

By the method of Example 13 we find that the work done by the steam in each cylinder per stroke on each square inch of the piston is

83·52 foot-pounds nearly,

and the work done by both pistons in one minute is

$$2 \times 58 \times \pi \times 81 \times 83{\cdot}52 = 2465382 \text{ foot-pounds.}$$

$$\therefore \text{H.-P.} = \frac{2465382}{33000} = 74{\cdot}7 \text{ nearly.}$$

(27.) The diameter of the piston in a non-expanding, double-acting engine is 16 inches. Its length of stroke is 4 feet 6 inches, and it makes 50 complete strokes per minute. The steam pressure is 17 lbs., and the mean value of the opposing pressure is $1\frac{1}{2}$ lb. to the square inch. What is the horse-power?

Pressure on piston

$$= \pi \times 8^2 (17 - 1\tfrac{1}{2}) = \pi \times 64 \times 15{\cdot}5 \text{ lbs.}$$

$\therefore$ work done per minute

$$= 2 \times 50 \times \pi \times 64 \times 15{\cdot}5 \times 4{\cdot}5,$$
$$= 1402410 \text{ foot-pounds.}$$

$$\therefore \text{H.-P.} = \frac{1402410}{33000} = 42\cdot5 \text{ nearly.}$$

(28.) The length of stroke of an engine is 5 feet, and the load is 20 lbs. to the square inch. If the steam is cut off when $\frac{1}{8}$th of the stroke has been completed, find the pressure at which the steam was admitted.

By the formula in Example 13 we find that

$$\text{P} = 62\cdot13 \text{ lbs. to the square inch.}$$

(29.) How much water must be evaporated per minute so as to supply the requisite quantity of steam to fill a cylinder 36 times a minute? The length of the cylinder is 5 feet, its diameter is 52 inches, and the volume of the steam is 1,240 times that of the water from which it is formed.

Volume of water required

$$= \pi \times \left(\frac{26}{12}\right)^2 \times \frac{5 \times 36}{1240} = 2\cdot141 \text{ cubic feet.}$$

(30.) The cylinder of an engine is 70 inches in diameter and the length of stroke is 7 feet. How much water must be evaporated per minute in order to fill this cylinder with steam 40 times, the volume of the steam at the pressure at which it is used being 1,400 times that of the water from which it is generated? *Answer.* 5·345 cubic feet.

MISCELLANEOUS EXERCISES.

CHIEFLY TAKEN FROM EXAMINATION PAPERS.

(1.) A Fahrenheit thermometer and a Centigrade thermometer when placed in different enclosures both indicate 72°. What is the difference between the temperatures of these two enclosures? *Answer.* $49\frac{7}{9}°$ C.

(2.) An iron yard measure is correct at the temperature of melting ice. What will be its error at the temperature of boiling water, the coefficient of expansion of iron being ·000012?

Answer. ·0432 of an inch too long. (Sc. and A.)

(3.) The coefficient of expansion of mercury is $\frac{1}{5550}$ and its density at 0° C. is 13·59. What will be the bulk of 30 kilogrammes of mercury at 85° C.? *Answer.* 2·24 litres.

(4.) What is the length of a bar of lead which expands as much for a given change of temperature as a bar of steel 3 metres long, the coefficient of expansion of lead being to that of steel as 351 : 927? *Answer.* 7·923 metres.

(5.) What must be the length of a bar of zinc which will expand as much for a given change of temperature as a bar of iron 2 metres long, the coefficient of expansion being to that of zinc as 340 : 795? *Answer.* 85·5 centimetres.

(6.) The ratio of the masses of equal bulks of water at 4° C. and of copper at 0° C. is 8·88, the coefficient of cubical expansion of copper is $\frac{1}{58200}$, and the expansion of unit volume of water between 4° and 15° C. is $\frac{1}{136}$. Find the density of copper with respect to water at 15° C. *Answer.* 8·943.

(7.) One hundred cubic centimetres of air at 0° C. are heated to 300° C. under constant pressure. What will be the volume of the air at the higher pressure, the coefficient of expansion being ·00366? *Answer.* 209·8 c.c. (M.)

(8.) The coefficient of expansion of atmospheric air for the Centigrade scale is $\frac{1}{273}$. Find the temperature to which 500 cubic centimetres of air (measured at 15° C.) must be raised in order that its volume may become 700 cubic centimetres, no change of pressure taking place meanwhile.

Answer. 130·2° C. (M.)

(9.) One gallon of air ($277\frac{1}{4}$ cubic inches) is heated under constant pressure from 0° C. to 75° C. Calculate the volume of the air at the latter temperature, assuming the coefficient of expansion to be ·00366.

Answer. 353·36 cubic inches nearly. (M.)

(10.) A room is calculated to hold 60 cubicmetres of air at 10° C. and 700 millimetres pressure. What would be the volume of this same quantity of air if it were measured at 0° C. and 760 millimetres pressure?

Answer. 53·31 cubic metres. (M.)

(11.) The coefficient of expansion of oxygen is $\frac{11}{3000}$, and 500 cubic centimetres of oxygen gas are measured when the temperature is 20° C., and the temperature is then raised to 40° C., the pressure meanwhile remaining invariable. What is the volume of the oxygen at the latter temperature?

Answer. 531 c.c. nearly. (M.)

(12.) A thousand cubic inches of air at the temperature of 30° C. are cooled down to zero, and at the same time the external pressure upon the air is doubled. What is its volume reduced to, the coefficient of expansion of air being ·00366?

Answer. 450·53 cubic inches. (M.)

(13.) A bottle is filled with air at the pressure of 76 centimetres of mercury and at the temperature of 25° C.; the bottle is securely corked and the temperature is reduced to 0° C. What will now be the pressure of the air?

Answer. 69·62 centimetres of mercury.

(14.) A Marriotte's tube has a uniform section of one square inch and is graduated in inches. Six cubic inches of air are enclosed in the shorter (closed) limb when the mercury is at the same level in both tubes. What volume of mercury must be poured into the longer limb in order to compress the air into two inches? *Answer.* 72 cubic inches. (1st B. Sc.)

(15.) Given the weight of one litre of dry air under the

normal conditions as 14·42 criths, what will be the weight of one litre of dry air at the normal temperature but under a pressure of 72 centimetres? *Answer.* 13·67 criths.

N.B. The *crith* is the quantity of hydrogen in one litre at 0° C. and 76 centimetres pressure.

(16.) The volume of nitrogen in a bell jar standing over a mercury pneumatic trough was 250 c.c. The barometer was at 75·4 centimetres, and the difference of level of the mercury inside and outside the jar was 6·5 centimetres. Reduce this volume to standard pressure. *Answer.* 226·7 c.c.

(17.) A volume of 64 cubic feet of air under a pressure of 29·4 inches of mercury and at a temperature of 15° C. is heated to a temperature of 100° C., and the pressure is increased to 30 inches. Find the resulting volume, the coefficient of expansion being $\frac{1}{273}$. *Answer.* 81·24 cubic feet. (Sc. and A.)

(18.) A closed vessel which displaces one litre of air is counterpoised on a balance with weights whose volume is inconsiderable when compared with that of the vessel. The balance is in equilibrium when the barometer stands at 76 centimetres. If the barometer fall to 71 centimetres, what weight must be added so as to restore the equilibrium?

Answer. 85 milligrammes.

(19.) A body is weighed in air and in water which are both at the temperature of 20° C. Determine its volume and specific gravity at this temperature from the following data:—

Weight of the body in air .	= 18·472 grammes.
Loss of weight in water . .	= 9·248 „
Weight of 1 c.c. of air at 0° C. and 76 centimetres . .	= ·001293 gramme.
Weight of 1 c.c. of water at 20° C.	= ·99827 „
Specific gravity of weights .	= 8·4.

Answers. Volume . = 9·274 c.c.
Specific gravity = 1·993.

(1st B. Sc. Hons.)

(20.) A solid weighs 320 grammes in vacuo, 240 grammes in distilled water at 4° C., and 242 grammes in water at 100° C. of which the density is 0·959. Find the volume of the solid at

these two temperatures, and deduce therefrom its mean coefficient of cubical expansion for 1° C. *Answer.* ·0001738.
(1st B. Sc.)

(21.) Suppose that the proportional cubical internal expansion of a glass specific gravity bottle between 0° C. and 100° C. is ·00235 while the similar expansion of mercury is ·018153. Suppose also that when the bottle contains a piece of iron weighing 2,000 grains the remainder of it will contain 6,707·8 grains of mercury at 0° C., while at 100° C. under these circumstances it will only contain 6,599·4 grains. Finally, assume that the specific gravities of mercury and iron at 100° C. are 13·2 and 7·8 respectively. Determine the cubical dilatation of iron between 0° C. and 100° C. *Answer.* ·00364.
(2nd B. Sc.)

(22.) A solid is weighed in a liquid at 0° C. and 100° C. The volume of the solid at 0° C. is unity and at 100° C. it is 1·006. Also the loss of weight by weighing in the liquid is, at 0° C. 1,800 grains, and at 100° C. it is 1,750 grains. Find the coefficient of dilatation of the liquid. *Answer.* ·0003474.
(1st B. Sc.)

(23.) Assuming that the mean coefficient of expansion of mercury for 1° C. is ·0001815 and that of the glass of a thermometer ·000026, find the reading of such a thermometer of which the bulb is plunged into water at the temperature of 100° C. while the stem is exposed to air at the temperature of 10° C.
Answer. 98·62. (2nd B. Sc.)

(24.) At what temperature does a litre of air weigh one gramme when the pressure is 77 centimetres, the coefficient of expansion of air being ·00366 and the mass of a litre of air at 0° C. and 76 centimetres pressure being 1·293 gramme?
Answer. 84·7° C.

(25.) The coefficient of linear expansion of iron for 1° C. being ·0000122, what will be the area of an iron disc at 60° C. whose diameter at 0° C. is 2·75 metres?
Answer. 5·9483 square metres.

(26.) Find the capacity of a flask which weighs 263·528 grammes empty, and 375·826 grammes when filled with air at 4° C. and 76 centimetres pressure. Also, when the flask is filled

with a certain gas at 4° C. and 81 centimetres pressure it weighs 293·687 grammes. Find the density of this gas.

Answer. Capacity of flask . . = 87·64 litres.
Absolute density of the gas = ·000323 gramme.

(27.) An English barometer with a brass scale giving true inches at the temperature of 62° F. reads 29·5 inches at 45° F. What is the pressure in true inches of mercury reduced to the specific gravity it has at 32° F.?

Coefficient of linear expansion of brass for 1° F. is ·00001.
" cubical " mercury " ·0001.

Answer. 29·445 inches. (1st B. Sc.)

(28.) A given quantity of air occupies a volume of 600 cubic inches at a temperature of 20° C. Find the volume which the air will occupy at 100° C., supposing the pressure to remain constant. The coefficient of expansion of air is ·003665.

Answer. 763·8 cubic inches. (Sc. and A.)

(29.) The coefficient of absolute expansion of mercury being $\frac{1}{5550}$ and its coefficient of apparent expansion being $\frac{1}{6480}$, find the coefficient of cubical expansion of glass.

Answer. $\frac{1}{386 \cdot 71}$ nearly. (Sc. and A.)

(30.) The diameter of a spherical hot-air balloon is 2·5 metres, and the paper of which it is made weighs 10 grammes per square metre. The temperature of the atmosphere is 7° C. and the pressure 76 centimetres. The coefficient of expansion of air being ·00366, what must be the temperature of the air in the balloon so that it may just be able to rise?

Answer. 12·4° C.

(31.) The mass of an empty flask is 247·862 grammes. When filled with air at 0° C. and 76 centimetres pressure it is 263·759 grammes, and when filled with another gas at 0° C. and 74 centimetres pressure it is 267·078 grammes. Find the relative density of this gas. (Air = 1.) *Answer.* 1·11.

(32.) An empty flask weighs 152·475 grammes. When full of dry air it weighs 168·386 grammes, and when filled with another gas at the same temperature and pressure it weighs 157·235 grammes. Find the density of this gas.

Answer. ·299 nearly.

(33.) What is the pressure of the atmosphere in grammes weight on each square centimetre of a surface when the barometer stands at 76 centimetres, the density of mercury being 13·596? *Answer.* 1,033 grammes.

(34.) To what difference of pressure does a difference of 1 centimetre in the barometric column correspond?

Answer. 13·596 grammes weight.

(35.) To what temperature must an open flask be heated before one quarter of the air which it contained at 0° C. is driven out? *Answer.* 91·07° C.

(36.) Find the pressure of the atmosphere upon a rectangular area whose diagonal is 44 centimetres and one side 26 centimetres, the temperature being 0° C. and the pressure 76 centimetres. *Answer.* 953·637 kilogrammes.

(37.) A glass flask contains 11·572 grammes of air at 0° C. and 75 centimetres pressure. It contains 12·317 grammes of a gas (A) at 0° C. and 79 centimetres pressure, and 7·221 grammes of a gas (B) at 0° C. and 70 centimetres pressure. Find the relative densities (air = 1) of these two gases.

Answer. Density of A = 1·01.
" B = ·66.

(38.) What is the hygrometric state in a room at temperature 20° C. in which the dew-point is found to be 11° C.?

Maximum tension of aqueous vapour at 20° C. . . = 17·39 millimetres.
Maximum tension of aqueous vapour at 11° C. . . = 9·79 "

Answer. ·54. (1st. B. Sc.)

(39.) If a cubic foot of dry air at 60° C. and 30 inches pressure be saturated with moisture, find what volume it will occupy, the pressure and temperature remaining the same. Maximum tension of aqueous vapour at 60° C. = 5·86 inches.

Answer. 1·2428 cubic feet.

(40.) In the eudiometrical analysis of a hydrocarbon gas, the following numbers were obtained. Find in each case the corrected volume of dry gas at 0° C. and 1 metre pressure.

	Observed Volume	*Temp. C.*	*Difference of Mercury level*	*Height of Barometer*
Gas used (moist) .	91·8 c.c.	12·8°	623·1 m.m.	752 7 m.m.
Gas after admission of oxygen (moist)	535·1 „	12·9°	160·6 „	751·7 „
Gas after combustion (moist) .	498·8 „	12·8°	194·0 „	751·1 „
Gas after absorption of CO_2 (dry) .	454·3 „	13·0°	237·2 „	741·5 „

The maximum pressures of aqueous vapour at the above temperatures are

at 12·8° C. . 11·0 m.ms. of mercury.
12·9° C. . 11·1 „ „

Answers. 1·04 c.c.; 295·14 c.c.; 259·58 c.c.; 218·18 c.c.

(41.) A kilogramme of water at 100° C. mixed with a kilogramme of melting snow without any loss of heat gives 2 kilogrammes of water at 10·3° C. Find from this the latent heat of water. *Answer.* 79·4 units. (M.)

(42.) A pound of ice, taken out of a mixture of ice and water, mixed with 5 pounds of water at 71° F., gives 6 pounds of water at 42°. Find how much the quantity of heat required to melt any quantity of ice at 32° would raise the temperature of the same quantity of water at 32°, assuming the specific heat of water constant. *Answer.* 135° F. (1st B. Sc.)

(43.) What will be the resulting temperature of the water when 3·5 kilogrammes of crushed ice at 0° C. are mixed with 45 kilogrammes of water at 32° C.? *Answer.* 23·98° C.

(44.) Into 8·5 kilogrammes of water at 89° C. there are plunged 1·75 kilogramme of crushed ice at 0° C. What will be the final temperature of the mixture, assuming that no heat is lost to the containing vessel and that the latent heat of water is 79? *Answer.* 47·93° C.

(45.) One kilogramme of ice at 0° C. and 3 kilogrammes of water at 79° C. are mixed in a closed vessel, the sides of which are supposed to be impervious to heat. The latent heat of water being 79, what will be the temperature of the water after the melting of the ice? *Answer.* 39·5° C. (M.)

(46.) If 6 kilogrammes of crushed ice at 0° C. are mixed with 28 kilogrammes of water at 40° C., and the latent heat of water is 79, find the resulting temperature.

Answer. 19° C.

(47.) How much steam at 100° C. is required to raise the temperature of 54 ounces of water from 0° C. to 100° C., the latent heat of steam being 540? *Answer.* 10 ounces.

(M.)

(48.) The heat produced by the complete combustion of 1 gramme of carbon in a calorimeter can convert 100 grammes of ice at 0° C. into water at 0° C. How many grammes of water could be raised by the same amount of heat from 0° C. to 1° C.? (Latent heat of water, 80.)

Answer. 8,000 grammes. (M.)

(49.) The latent heat of steam being 536, what will be the resulting temperature of 20 litres of water, initially at 4° C., when one kilogramme of steam at 100° C. has been condensed in it? *Answer.* 34·1° C.

(50.) How much water at 45° C. must be mixed with 11 kilogrammes of crushed ice so that the temperature of the mixture may be 12° C.? The latent heat of water is assumed to be 79. *Answer.* 30·333 kilogrammes.

(51.) What will be the final temperature of the water resulting from the mixture of 8 pounds of crushed ice at 0° C. with 35 pounds of water at 59° C.? *Answer.* 33·3° C.

(52.) How many pounds of water at 45° C. are required in order to reduce 8 pounds of ice to water at 0° C., the latent heat of water being 79? *Answer.* 14·044 pounds.

(53.) Find the mass of a cubic metre of air at 30° C. and 77 centimetres pressure from the following data:—

Mass of one litre of air at 0° C. and 76 centimetres pressure .	= 1·293 grammes.
Hygrometric state of the air .	= ·75
Maximum tension of aqueous vapour at 30° C.	= 3·45 centimetres.
Density of aqueous vapour (air = 1)	= $\frac{5}{8}$

Answer. 1,167 grammes.

(54.) If 8 ounces of zinc at 95° C. be put into 20 ounces of

water at 15° C. and the resulting temperature be 18° C., what is the specific heat of zinc? *Answer.* ·0974. (Sc. and A.)

(55.) The specific heat of water is 30 times as great as that of mercury. If a pound of boiling water be mixed with a pound of ice-cold mercury, what will be the final temperature of the mixture? *Answer.* 96·8° C. (Sc. and A.)

(56.) The specific heat of water is $10\frac{1}{2}$ times that of copper. If 15 pounds of copper at 80° C. be immersed in 18 pounds of water at 42° C., find the temperature to which the water will rise. *Answer.* 44·8° C. (Sc. and A.)

(57.) How many pounds of crushed ice are required to reduce 25 pounds of steam at 100° C. to water at 0° C., the latent heat of water being 79·25 and that of steam 540?

Answer. 201·892 pounds.

(58.) When a pound of water at 0° C. is mixed with a pound of mercury at 100° C. the temperature of the mixture is 3° C. Find the thermal capacity of mercury. *Answer.* $\frac{3}{97}$.

(59.) Compare the quantity of heat which is required in order to heat one pound of water from 0° C. to 1° C. with the quantity which is required in order to convert one pound of ice at 0° C. into steam at 100° C. (Latent heat of water = 79·25; latent heat of steam = 536.) *Answer.* $\frac{4}{2861}$. (M.)

(60.) How much ice at 0° C. can be converted into water at 0° C. by an ounce of steam, if we assume heat to be transmitted from the steam only to the ice? (Latent heat of water = 80; latent heat of steam = 536.) *Answer.* 7·95 ounces. (M.)

(61.) A ball of platinum whose mass is 200 grammes is removed from a furnace and immersed in 150 grammes of water at 0° C. If we suppose the water to gain all the heat which the platinum loses, and if the temperature of this water rises to 30° C., what is the temperature of the furnace? (Specific heat of platinum = ·031.) *Answer.* 755·8° C. (Sc. and A.)

(62.) A metal calorimeter whose mass is 40 pounds and the specific heat of its material being ·12 contains 32·5 pounds of water at 11·5° C. When 8·25 pounds of another metal at 60·5° C. are immersed in the water the resulting temperature is 14·6° C. Find the specific heat of this metal. *Answer.* ·305.

(63.) Steam enters the condenser at a temperature of 212° F. and the water pumped out of the condenser is at a temperature

of 110° F., while the temperature of the injection water is 60° F. What quantity of injection water must be supplied for each pound of steam which enters the condenser, the latent heat of steam at 212° F. being 966·6?

Answer. 21·17 pounds nearly. (Sc. and A.)

(64.) The specific heat of zinc is ·095, and 280 grammes of zinc are raised to the temperature of 97° C. and immersed in 150 grammes of water at 14° C., contained in a copper calorimeter weighing 96 grammes, the specific heat of copper being ·095. What will be the temperature of the mixture, supposing that there is no exchange of heat except among the substances mentioned? What is the water equivalent of the calorimeter employed?

Answers. Final temperature = 25·89° C.
Water-equivalent = 9·12 grammes.
(1st B. Sc.)

(65.) A square metre of a substance 1 centimetre thick has one side kept at 100° C. and the other, by means of ice, at 0° C. In the course of 30 minutes one kilogramme of ice is melted by this operation. Taking the latent heat of water at 79, find the conductivity of this substance in centimetre-gramme-minute units. *Answer.* ·00263. (1st B. Sc.)

(66.) A metal plate a quarter of an inch in thickness and 2 feet square has the whole of one face in contact with water which is kept boiling, while the other face is in contact with melting ice, and it is found that 300 pounds of ice are melted in one hour. Find the absolute conductivity of the metal in inch-pound-minute units. *Answer.* $\frac{5}{48}$. (M.)

(67.) A man whose weight is 16 stone walks up a staircase 100 feet high. How much heat does he expend in doing this?

Answer. 16·115 w.-p.-d. C.°

(68.) Find the distance through which a mass of 10 pounds can be raised against gravity by the expenditure of the quantity of heat which would be sufficient to raise one pound of water at 0° C. to 5° C. *Answer.* 695 feet.

(69.) A 16-pound cannon-ball is stopped by a wall when it is moving with a velocity of 2,500 feet per second. What quantity of heat will be produced by the collision?

Answer. 1117·1 w.-p.-d. C.°

FIG. 1.

FIG. 2.

FIG. 3.

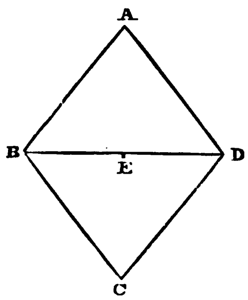

FIG. 4.

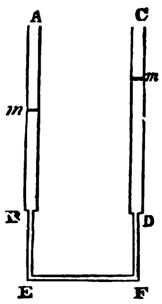

FIG. 5.